

# Global Sensitivity Analysis Applied to Electric Motor Design by Polynomial Chaos Expansion

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The design of electrical motors is becoming more and more complex as it must deal with multi-physical analysis and multi-objective optimization. Using a surrogate model can enhance the exploration capabilities of optimization. Global Sensitivity Analysis is a technique that highlights the most critical input parameters influencing the performance of a device. The analysis of a V-shaped Interior Permanent Magnet (IPM) motor structure is attempted. Due to the computational intensity of performance evaluation, a data-driven model with 8 parameters generating 2 outputs (torque and temperature) is created using an artificial neural network. Polynomial Chaos Expansion is then applied on the surrogate model to assess the performance variation. Some results on a motor configuration are presented, and their potential in the design process is outlined.

**Index Terms**—Electric motor design, Global Sensitivity Analysis, Polynomial Chaos Expansion, Surrogate model

## I. INTRODUCTION

**S**URROGATE MODELS or meta-models in engineering are mainly aimed at getting more information out of analysis code runs. While the single run can be computationally expensive and can give information about a particular configuration, often what is needed by the designer is a *landscape* of the quantity of interest with respect to some input parameters. Traditionally, surrogate modeling was mostly used as a *curve fitting* interpolating between configurations that are close in parameters space. The outstanding advantage of the surrogate is that, once built, it is much more computationally efficient than the original analysis tool, allowing to explore regions that were not covered, even if its accuracy has to be validated.

However, when meta-modeling is used together with optimization procedures, the accuracy of the surrogate should be checked iteratively since the optimization search often moves the promising point far away from the regions that have been used to build it, straining the *curve fitting* hypothesis and possibly reducing its accuracy dramatically.

In this respect, the use of machine learning based surrogate models can help the accuracy of the surrogate model: while interpolation techniques have good behavior on existing data, machine learning based procedures are more focused on forecasting and looking for patterns in large-dimensional data, and they usually show good performance on data never seen before.

The availability of a robust surrogate model opens the door to new design methodologies like the Global Sensitivity Analysis (GSA) and the possibility of evaluating the propagating of the uncertainties in input parameters through the computer model.

In this contribution, an artificial neural surrogate model trained on the analysis datasets of an electrical motor is used to assess the GSA of the design problem. The parameter space consists of eight geometrical and supply parameters, and the outputs are multi-physical motor performance indicators, such as torque, torque ripple, and maximum temperature in overload conditions. [1].

TABLE I  
VARIABLES SWEEP LOWER AND UPPER RANGES

|   | Parameter           | name      | range            |
|---|---------------------|-----------|------------------|
| 1 | Barrier position    | $d\alpha$ | [0.65 0.85] (pu) |
| 2 | Barrier width       | $h_c$     | [0.3 0.7] (pu)   |
| 3 | Rotor radius        | $r$       | [60 78] (mm)     |
| 4 | Tooth width         | $w_t$     | [3.8 6.3] (mm)   |
| 5 | Tooth length        | $l_t$     | [15 22.5] (mm)   |
| 6 | Slot opening        | $w_o$     | [0.1 0.4] (pu)   |
| 7 | Barrier shift       | $dxIB$    | [-4 6] (mm)      |
| 8 | Current phase angle | $\gamma$  | [30 60] (°)      |

## II. SURROGATE MODEL BUILDING

### 1) dataset creation

Sampling is a fundamental step in surrogate model building, especially in high dimensions, as the sampled points are the hinges of the surrogate. Grid sampling systematically divides the space into equal intervals, ensuring regular coverage. However, its practicality decreases in high-dimensional spaces due to the exponential growth of the required number of points, a challenge known as the curse of dimensionality [2]. Sobol-based quasi-random sampling, by contrast, uses deterministic low-discrepancy sequences that distribute points more uniformly across the space [3]. These sequences work by iteratively refining the placement of points through a mathematical process that ensures the points are spread evenly, thereby reducing the risk of large gaps or clusters. This method offers superior coverage compared to uniform distributions. The Sobol sampling of the design parameter space allows to compute a dataset of points belonging to a  $p$  dimensional real space  $\mathbf{x} \in \mathcal{R}^p$  and to obtain a corresponding set of performance values by some analysis technique like finite element method or others  $\mathbf{y} \in \mathcal{R}^m$ . In this work, Sobol sequences are used to create a series of points in the design space, whose parameters are summarized in Table I. Datasets made up of 4096 points are used for the forthcoming computations.

### 2) artificial neural network surrogate

The nonlinear input-output relationship is approximated by an artificial neural network [4]. The structure of the network

TABLE II  
SURROGATE MODEL ERRORS ON THE SAMPLING POINTS.

|         | Torque<br>error (%) | Temperature<br>error (%) |
|---------|---------------------|--------------------------|
| Mean    | 0.534               | 0.041                    |
| 95 perc | 1.526               | 0.103                    |
| Max     | 6.725               | 0.869                    |

is different for each model output. A search among the neural network structures (number of layers, number of neurons per layer) minimizing the reconstruction error is carried out, and the best results are reported in Table II. The optimal network for torque computation is made up of 3 layers, each consisting of (24, 64, 1) neurons, leading to 1881 trainable parameters. For temperature, the optimal network consists of 4 layers, with (8, 16, 32, 1), totaling 793 trainable parameters.

### III. POLYNOMIAL CHAOS EXPANSION

Polynomial Chaos Expansion (PCE) is a non-sampling based method to determine the evolution of uncertainty in a nonlinear system when there is probabilistic uncertainty in the system parameters [5]. In this work an open source Python library is used. References can be found in [6]. Given a variable  $X(\omega)$  with finite variance where  $\omega$  describes the randomness of the parameter  $X$ , the influence of  $\omega$  through the nonlinear model can be assessed by an iterative sampling as in the Monte Carlo process. Instead, in PCE it is evaluated by a spectral expansion standing on a Hermite polynomial basis. Different distributions (normal, uniform, etc.) lead to different polynomial bases. The PCE analysis is then based on the following steps:

- 1) Polynomial selection: once the variable (parameter) random distribution is known, the related polynomial type is chosen. In this case, a uniform distribution is assumed. Thus, the Legendre polynomials are selected.
- 2) PCE accuracy order: define the *order*  $d$  for the polynomial expansion and all the combinations that lead to the multivariate polynomials.
- 3) PCE coefficients computation: calculate the PCE coefficients through the projection method by numerically integrating the performance function multiplied times the polynomials belonging to the basis. The higher the *level* of the numerical integration rule, the higher the number of function evaluations.
- 4) PCE Sensitivity indices: once the PC Model Output is obtained, mean and standard deviation values can be computed as a whole and as sensitivity functions with respect to each single parameter.

Considering the motor surrogate model previously defined, the sensitivity indices of torque and temperature with respect to the input variable can be assessed and are reported in Fig. (1). As it can be seen, the PCE model is able to assess the most critical model parameters. For instance, when temperature is concerned, parameters 4 and 5 defining the stator slot area are mostly influential. The convergence speed of the computation of sensitivity indices is shown in Fig. (2) where the  $S_3$  value is shown as a function both of the *order* of polynomial used and

of the number of function calls, that is of the *level* of numerical integration rule.

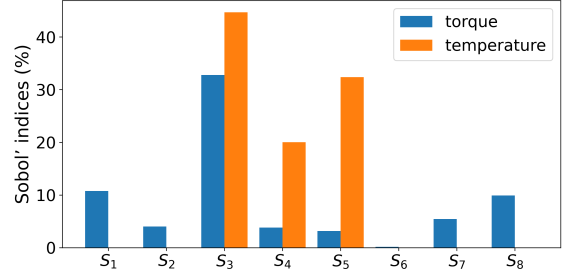


Fig. 1. Bar graph of the 8 sensitivity indices computed by PCE.

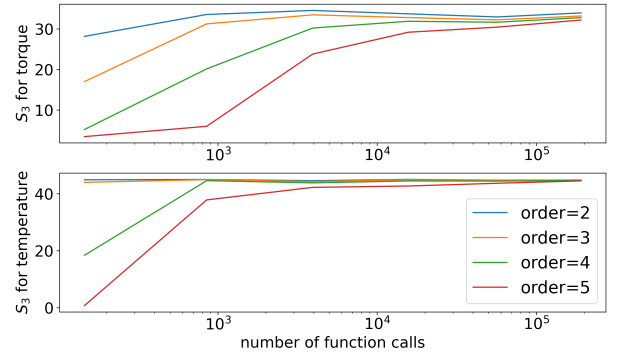


Fig. 2. Convergence speed of the computation of sensitivity indices  $S_3$ .

### IV. DISCUSSION AND FUTURE DEVELOPMENTS

The preliminary results obtained are encouraging in the definition of meta-modeling. Using a multi-objective optimization, the Pareto front of the design is evaluated, and then the PCE technique can be used to assess the uncertainty of the configurations belonging to the Pareto Front. This issue, particularly important in a surrogate assisted multi-objective optimization process, will be presented at the conference.

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