

Electro-thermal topology optimization of an electric machine by the topological derivative considering drive cycles

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We consider a 2d permanent magnet synchronous machine operating in a sequence of static operating points coming from a drive cycle. We aim to find a rotor design which maximizes the efficiency defined as the quotient of input and output energy considering Joule losses in the stator and eddy current losses in the permanent magnets. A coupled thermal analysis of the rotor considers the eddy current losses as heat source and adds a temperature constraint to avoid damage of the permanent magnets. To solve the resulting freeform topology optimization problem we use a level set description of the design and the topological derivative as sensitivity information. We show the effect of this constraint at very high speeds which is a trend in recent machine development.

Index Terms—Design optimization, drive cycles, multi-material topology optimization, thermal analysis

I. INTRODUCTION

DESIGN optimization of permanent magnet (PM) synchronous machines gives answers to the increasing demand for electric machines with high energy density and high efficiency [1]. With increasing rotation speed, eddy current losses become more and more important, especially since the PMs are very sensitive to high temperatures. We optimize the performance of a machine following a drive cycle, see, e.g., in [2], and present a method how to constrain the maximal rotor temperature while maintaining a high efficiency. Our 2d model machine is presented in Fig. 1 and Tab. I.

II. TOPOLOGY OPTIMIZATION FRAMEWORK

In [3], the authors maximized the average torque for a single operating point (OP) with respect to a maximum temperature constraint. Similarly we aim to optimize the rotor design Ω consisting of iron, air and permanent magnets with $M = 2$ different magnetization directions φ_1, φ_2

$$\Omega = (\Omega_f, \Omega_{m_1}, \dots, \Omega_{m_M}, \Omega_a),$$

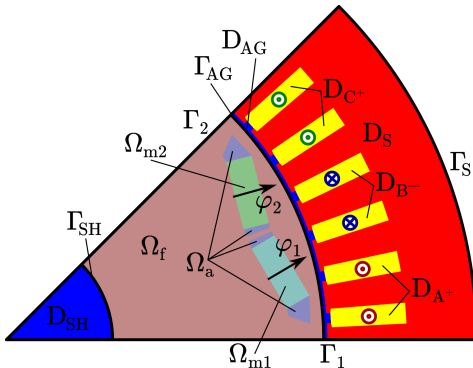


Fig. 1: One pole of the machine, the computational domain D_{all} consisting of stator iron D_S , coils D_{A+}, D_{B-}, D_{C+} , air gap D_{AG} , rotor design domain D (grayed) and shaft D_{SH} .

Number of poles and slots	8, 48
Inner and outer radius rotor	17.7mm, 52.4mm
Inner and outer radius stator	52.8mm, 77.3mm
Axial length ℓ_z	100mm
Stator resistivity R_S	3.2Ω
Number of turns per slot, fill factor	60, 0.6
Magnetization direction φ_1, φ_2	30°, 15°
Remanent flux density B_R	1.216T
Electric conductivity of magnet σ_m	6.7×10^5 S/m
Thermal conductivities $\lambda_f, \lambda_m, \lambda_a$	16, 9, 0.05 W/(mK)
Thermal robin transfer coefficients h_{SH}, h_{AG}	0.235, 30 W/(m²K)
Ambient temperature θ_0	40°C

TABLE I: Machine geometry and material data

where $\Omega_f, \Omega_{m_1}, \dots, \Omega_{m_M}, \Omega_a$ form a partition of the design domain D . In our framework the design Ω is described by a vector-valued level set function and updated by the topological derivative, i.e., the sensitivity of a cost functional $\mathcal{J}$ to point-wise material changes. One applies an adjoint method to compute this sensitivity. Compared to the widely used density approach, this method provides sharp material interfaces and enables arbitrary topology changes [1]. A drawback is the additional computational cost that comes with the computation of the topological derivative. However, as outlined in [3], one can move the expensive part to an offline phase computing samples which are interpolated during the optimization.

III. DRIVE CYCLE ANALYSIS

We consider the motoric part of the WLTP3 drive cycle [4], i.e. the OPs with positive torque. In order to reduce the number of OPs we chose $K = 30$ cells of operation represented by an OP each and measured the relative active time t_k for each cell by counting the original OPs in this range. Our goal is to maximize the efficiency

$$\mathcal{E}(\Omega) = \frac{\sum_{k=1}^K t_k \omega_k T_k}{\sum_{k=1}^K t_k (\omega_k T_k + P_k^J(\Omega) + P_k^{EC}(\Omega))}, \quad (1)$$

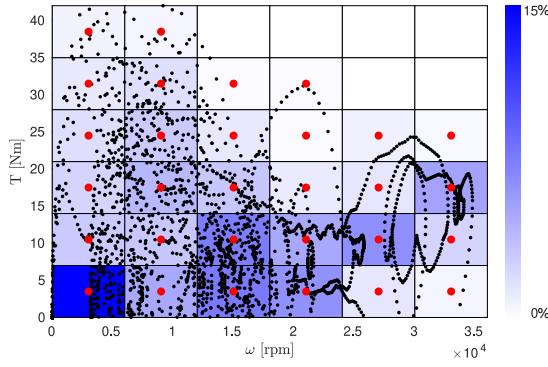


Fig. 2: WLT3 drive cycle. Relative active time t_k as heat map. Representative OPs (T_k, ω_k) of selected cells in red.

with Joule losses $P_k^J(\Omega) = 3R_S I_k(\Omega)^2$ in the coils and eddy current losses in the PMs P_k^{EC} (to be defined later) with respect to a volume constraint on the PMs. To determine the root mean square source current I_k and current angle β_k for each OP we solve the maximal torque per ampere (MTPA) problem

$$(I_k(\Omega), \beta_k(\Omega)) = \underset{(I, \beta)}{\operatorname{argmin}} I \text{ s.t. } \bar{\mathcal{T}}(\Omega, (I, \beta)) = T_k, \quad (2)$$

where $\bar{\mathcal{T}}$ is the average torque over N rotor positions depending on the current rotor design Ω and source current parameters (I, β) . We maximize the efficiency $\mathcal{E}$ by maximizing the weighted sum of torques $\mathcal{J}(\Omega) = \sum_{k=1}^K t_k \omega_k \bar{\mathcal{T}}(\Omega, (I_k, \beta_k))$ for fixed input currents I_k, β_k updating them every N_{MTPA} iterations by solving the MTPA problem (2).

IV. TEMPERATURE CONSTRAINT

For each OP k we compute the eddy current losses q_k^{PM} in the PMs in a postprocessing based on the third component of the magnetic vector potential a_k^n for the n -th rotor position resulting from magnetostatic simulations according to [5], assuming that the PMs maintain their topological structure throughout the optimization. Ignoring transient thermal effects, we solve a static heat equation to obtain the temperature distribution θ_k

$$\begin{aligned} q_k^{PM} &= \frac{\sigma_m}{N \tau_k^2} \sum_{n=1}^N \sum_{i=1}^M \chi_{\Omega_{m_i}} \left(a_k^{n+1} - a_k^n - \overline{a_k^{n+1} - a_k^n}^i \right)^2 \\ -\operatorname{div} \lambda_{\Omega} \nabla \theta_k &= q_k^{PM} \quad \text{in } D \\ \lambda_{\Omega} \nabla \theta_k \cdot n_{AG} &= h_{AG}(\theta_k - \theta_0) \quad \text{on } \Gamma_{AG} \\ \lambda_{\Omega} \nabla \theta_k \cdot n_{SH} &= h_{SH}(\theta_k - \theta_0) \quad \text{on } \Gamma_{SH}, \end{aligned}$$

where $\bar{v}^i = |\Omega_{m_i}|^{-1} \int_{\Omega_{m_i}} v \, dx$ is the spatial average of v over the i -th PM. Note that $q_k^{PM} \sim \omega_k^2$ via the time step size τ_k . We add a penalty term to our cost functional to enforce a point-wise temperature constraint $\theta_k \leq \theta^* = 90^\circ$ for all OPs

$$\mathcal{J}(\Omega) = \sum_{k=1}^K t_k \left(\omega_k \bar{\mathcal{T}}(\Omega, (I_k, \beta_k)) + 10^8 \int_D \Phi(\theta_k) \, dx \right),$$

where $\Phi(\theta) = \max\{0, \theta - \theta^*\}$ is chosen such that the latter integral vanishes if the constraint is fulfilled everywhere in D [6]. Therefore, adding it to the objective with a sufficiently

high weight yields an equivalent unconstrained problem. The total eddy current losses for the efficiency (1) are computed by $P_k^{EC}(\Omega) = \ell_z \int_{\Omega_{m_1} \cup \Omega_{m_2}} q_k^{PM} \, dx$.

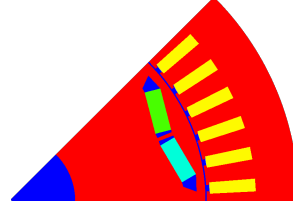


Fig. 3: Initial design Ω_{init}

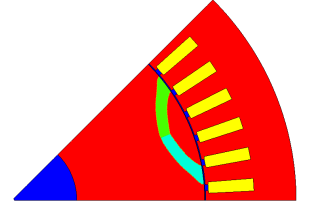


Fig. 4: Final design without temperature constraint Ω^*

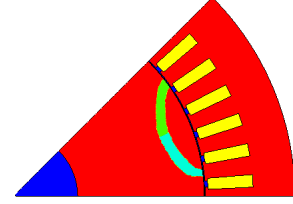


Fig. 5: Final design with temperature constraint Ω_{θ}^*

	$\mathcal{E}$	θ_{max}
Ω_{init}	90.64%	43°
Ω^*	94.76%	128°
Ω_{θ}^*	94.66%	83°

TABLE II: Efficiency and maximal temperature of obtained designs

V. CONCLUSION & OUTLOOK

As presented in Tab. II we succeed in increasing the efficiency from the initial design Ω_{init} (Fig. 3) to Ω^* (Fig. 4) for the fixed magnet volume which comes with a damaging increase of temperature in the rotor. By adding the temperature constraint the resulting design Ω_{θ}^* (Fig. 5) almost maintains the efficiency but does not overheat anymore.

The usage of a static heat equation is based on the assumption that we stay long in each OP which is a conservative assumption. One could improve this prediction by considering a transient heat equation with a piece-wise-in-time constant heat source. Further one can improve the accuracy of the eddy-current losses by solving the transient magnetoquasistatic equation, as it was done in [3], and consider additionally iron losses. At such high speeds, it will also be relevant to consider mechanical constraints. Another interesting extension is to optimize the orientations of magnetization φ_1, φ_2 simultaneously.

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