

Worst-case expectation approach to a Quadrupole Resonator optimization under uncertainty

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The investigation of the fundamental properties of materials such as niobium or Nb₃Sn by high-precision surface resistance measurements is a crucial aspect for the further development of superconducting radio-frequency (RF) technology. A calorimetric method using the dedicated quadrupole resonator (QPR) instrument is now one of the most accurate and reliable measurement methods for determining the high-frequency properties of superconducting materials with a resolution in the nΩ range. As with other resonators, the QPR deviates from its nominal design during operation for various reasons. These shape deviations can be static, e.g., due to manufacturing variations reflected in the manufacturing tolerances, or dynamic, e.g., because of physical phenomena such as Lorentz force (LF) detuning or microphonics. This, in turn, can significantly limit the measurement accuracy and general operation of a QPR. In our work, therefore, we developed an algorithm for multi-objective and multi-physics optimization that contributes to estimating shape deviations. The optimized QPR design is insensitive to geometrical uncertainties and compensates for the effects of LF detuning and high-frequency losses at the adapter flange.

Index Terms—quadrupole resonator, worst-case expectation analysis, multi-objective and multi-physics optimization, Lorentz force detuning, microphonics.

I. INTRODUCTION

In particle accelerator technology, current RF resonators are often made of niobium and operated at ultra-low temperatures in the superconducting state to minimize surface losses. For future accelerator structures, new superconducting materials or thin-film deposition techniques can further reduce surface losses and associated operating costs. To accurately determine the RF properties of such superconducting materials, a calorimetric measurement is performed using a QPR device. The QPR instrument was originally developed at CERN in 1997 by [1] to conduct high-precision surface resistance measurements on niobium samples. Later, its design was adapted to operating frequencies of 433 MHz, 866 MHz, and 1.3 GHz at the Helmholtz-Zentrum Berlin [2] and recently modified further at the University of Hamburg [4] in cooperation with the Deutsches Elektronen-Synchrotron DESY.

The QPR device, as shown in Fig.1, consists of a screening cavity made of two thermally isolated chambers and four rods, which are welded at the top plate and bent at the end into half-ring pole shoes. The calorimetry chamber is located below these two loops. The power loss caused by eddy currents is measured by temperature sensors installed inside the calorimetry chamber as $\Delta P_{RF}(\mathbf{p}) := [P_{DC1}(\mathbf{p}) - P_{DC2}(\mathbf{p})]$. Here, P_{DC1} and P_{DC2} denote the heating power required for temperature stabilization at the initial temperature T_{int} or the reduced heating power P_{DC2} , which is determined after the RF antenna is switched on and the equilibrium for T_{int} is reached again. Finally, when using the calorimetric “RF-DC compensation” method, the surface resistance is calculated by

$$R_S(\mathbf{p}) \doteq \frac{2 \Delta P_{RF}(\mathbf{p})}{\int_{\Omega_S} \|\mathbf{h}(\mathbf{p})\|^2 d\mathbf{x}}, \quad (1)$$

where $\mathbf{h}(\mathbf{p})$ denotes the magnetic field strength as a function of geometrical parameters, while the denominator is the

product of a simulation constant c and the energy U stored in the cavity [1]. As a result, the surface resistance is prone to various uncertainties associated not only with the “RF-DC-compensation” method. In practice, the QPR shape may deviate from its nominal design due to manufacturing imperfections or dynamic physical phenomena associated with electromagnetic radiation pressure or microphonics [3], [5].

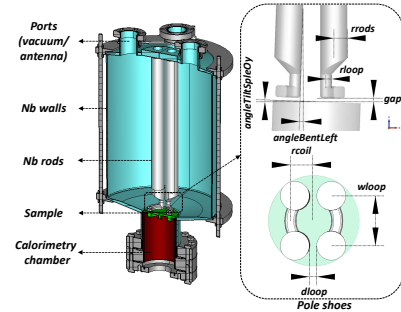


Fig. 1. Layout of the QPR (left) with its parameterized model (right) [5].

Our new contribution is a multi-objective (MO) and multi-physics QPR optimization under geometric uncertainties, which requires modeling physical phenomena such as LF detuning, microphonics, and industrial imperfections. For this reason, we employ the worst-case expectation analysis of the multi-physics problem to efficiently model a nonlinear deformation of the QPR in a quasi-static regime using the polynomial chaos stochastic collocation method (PC-SCM), [7].

II. PHYSICAL MODEL

We address a multi-physics cavity problem governed by the stochastic, weakly coupled electromagnetic-stress-heat problem, which involves the expectation of the weak form, $\mathbb{E}[\cdot] = \langle \cdot, \cdot \rangle_{L^2_p}$, formulated in a physical space: Find $(\lambda, \mathbf{h}) \in \mathbb{R}^+ \times V_\rho$

and $\mathbf{h} \neq 0$, $\mathbf{u}, T \in V_\rho = H_0^1(D) \otimes L_\rho^2(\Gamma)$ such that

$$\mathbb{E} \left[\langle \mu_r(\theta)^{-1} \nabla \times \mathbf{h}, \nabla \times \boldsymbol{\varphi} \rangle_{D_c} - \lambda(\theta) \langle \epsilon_r(\theta) \mathbf{h}, \boldsymbol{\varphi} \rangle_{D_c} \right] = 0, \quad (2a)$$

$$\mathbb{E} [\langle \mathbf{C} \cdot \boldsymbol{\varepsilon}(\mathbf{u}), \nabla \mathbf{v} \rangle_{D_w}] = 0, \quad (2b)$$

$$\mathbb{E} \left[\langle \kappa(\theta) \nabla T, \nabla \vartheta \rangle_{D_w} - \left\langle \frac{\gamma(\theta)}{\epsilon(\theta) \omega(\theta)} |\nabla \times \mathbf{h}|^2, \vartheta \right\rangle_{D_w} \right] = 0, \quad (2c)$$

imposed with the LF pressure on the rods boundary $\Gamma^* = \partial D_{cw}$

$$\mathbb{E} [\langle \boldsymbol{\sigma}, \mathbf{n}_w \rangle_{\Gamma^*}] = \mathbb{E} \left[\frac{1}{4} \left\langle \frac{1}{\omega(\theta)} |\nabla \times \mathbf{h}|^2 + \mu(\theta) |\mathbf{h}|^2, \mathbf{n}_w \right\rangle_{\Gamma^*} \right] \quad (2d)$$

for all test functions $\boldsymbol{\varphi}, \mathbf{v}, \vartheta \in V_\rho$, where $\theta = (\mathbf{x}, f, \mathbf{p}) \in D \times \mathcal{F} \times \Pi$ with the computational domain D , the frequency range $\mathcal{F}$ and a multidimensional parameter domain Π , respectively. In (2), $\omega(\theta) = 2\pi f \in \mathbb{R}^+$, $\mu(\theta) \in L^\infty(D)$, and $\epsilon(\theta) \in L^\infty(D)$ denote the angular frequency with the frequency f , the magnetic permeability, and the complex electric permittivity. The thermal conductivity and the electric conductivity are assumed to be real-valued functions such as $\kappa(\theta) \in L^\infty(D)$ and $\gamma(\theta) \in L^\infty(D)$. Hooke's law relating the linearized Cauchy-Green strain tensor and the Cauchy stress tensor $\mathbf{C}(\theta)$ is assumed to be independent of the displacement $\mathbf{u} = \mathbf{u}(\theta)$. Specifically, $\boldsymbol{\sigma} = \mathbf{C} \cdot \boldsymbol{\varepsilon}(\mathbf{u})$. Finally, the temperature is denoted by $T = T(\theta)$.

III. WORST-CASE-EXPECTATION ANALYSIS

For the uncertainty quantification (UQ), let the vector $\mathbf{p} \in \Pi \subset \mathbb{R}^Q$ represent Q certain parameters, shown in Fig.1, which are replaced by normally distributed variables. Then, the truncated PC expansion model of ψ_j with I terms reads as

$$\psi_j(\mathbf{x}, f; \mathbf{p}) \doteq \sum_{i=0}^I \alpha_i(\mathbf{x}, f) \Phi_i(\mathbf{p}), \quad j = 1, \dots, N \quad (3)$$

with unknown coefficient functions α_i and Hermite polynomials as basis $\Phi(\mathbf{p})$, corresponding to a Gaussian distribution. Next, we combine the PC-SCM and the perturbation shape analysis to approximate the worst-case expectation bound [7]

$$\Delta F_j[\Omega(\bar{\mathbf{p}})] = \sup_{\Omega(\mathbf{p}) \in \Pi} \mathbb{E} [\psi_j(\Omega(\mathbf{p})) - \psi_j(\Omega(\mathbf{p}_0))], \quad (4)$$

with the merit function $\psi_j(\Omega(\bar{\mathbf{p}})) = F_j[\Omega(\bar{\mathbf{p}}), \mathbf{h}(\Omega(\bar{\mathbf{p}}))]$, where $\Omega(\mathbf{p}) = \left\{ \mathbf{p} \in \Pi \subseteq \mathbb{R}^Q \mid \mathbf{0} \leq \mathbf{p}_{\min} \leq \mathbf{p} \leq \mathbf{p}_{\max} \right\}$ with inequalities understood component-wise.

IV. MO MULTI-PHYSICS OPTIMIZATION

Now, we use a robust formulation of the stochastic merit functions $F_j(\Omega(\mathbf{p}), \cdot)$ with the prescribed parameter $0 \leq \beta \leq 1$

$$F_j(\Omega(\mathbf{p}); \mathbf{h}) := \beta \mathbb{E}[\psi_j(\cdot)] + (1 - \beta) \sqrt{\text{Var}[\psi_j(\cdot)]} \quad (5)$$

to conduct the MO, multi-physics optimization of the QPR

$$\inf_{\Omega(\bar{\mathbf{p}}) \in \Xi} \mathcal{J}(\Omega(\mathbf{p}); \mathbf{h}) = \left[F_1(\cdot), F_2(\cdot), F_3(\cdot) \right]^\top, \quad (6a)$$

$$\text{s.t.} \quad \begin{cases} e_1(\Omega(\mathbf{p}), [\lambda, \mathbf{h}]), & (2a) \\ e_2(\Omega(\mathbf{p}), [\mathbf{u}, \mathbf{h}]), & (2b) \\ e_3(\Omega(\mathbf{p}), [T, \mathbf{h}]), & (2c) \end{cases}$$

with $\sqrt{\text{Var}(\psi_j)}$ approximated by the linear term of the bound (4). To solve the problem in (6), we apply the steepest descent method with Lagrange multipliers and variance-based sensitivity as a robust gradient approach [5], [6].

V. NUMERICAL EXAMPLE & DISCUSSION

In the MO and multi-physics optimization, the parameters listed in Tab. I and shown in Fig. 1 are considered as random design parameters, $Q = 11$, which are modeled by a Gaussian distribution such that $\sigma(p_i) = 0.05$ mm with Hermite polynomials of order $d = 2$ in the truncated PC model (3). To approximate (4), the first-order Taylor expansion with the perturbation shape analysis around the mean design parameter $\bar{\mathbf{p}}$ is used [7]. As objective functionals, we chose the focusing factor $F_1(\cdot)$, the homogeneity factor $F_2(\cdot)$ and the dimensionless factor $F_3(\cdot)$, calculated in the area of sample Ω_S and flange Ω_F . We also considered the maximum of the displacement magnitude, $F_4(\cdot)$, in the area of the pole shoes $\Omega_{PS} \subset \Omega_R$, using the method of Lagrange multipliers [5]. The preliminary results are presented in Tabs I and II, respectively.

TABLE I
RESULTS FOR MO OPTIMIZATION – PARAMETER DOMAIN.

Name		$\Omega_{\text{HVB}}^*(\bar{\mathbf{p}})$ [3]	$\Omega_{\text{Sol.D}}^*(\bar{\mathbf{p}})$
p_1 (gap)	mm	0.50	0.57
p_2 (rrods)	mm	13.00	9.42
p_3 (hloop)	mm	10.00	9.73
p_4 (rloop)	mm	5.00	5.11
p_5 (wloop)	mm	44.00	43.74
p_6 (dloop)	mm	6.00	4.02
p_7 (rcoil)	mm	22.408	24.73
p_8 (rsample)	mm	37.50	35.49
p_9 (ltrans1)	mm	7.50	5.23
p_{10} (drrods)	mm	2.00	2.55
p_{11} (ltrans2)	mm	15.00	13.62

TABLE II
RESULTS FOR MO OPTIMIZATION – OBJECTIVE SPACE*.

Means/Configurations	$\Omega_{\text{HVB}}^*(\bar{\mathbf{p}})$	$\Omega_{\text{Sol.D}}^*(\bar{\mathbf{p}})$	[%]
$^a F_1(\Omega^*(\bar{\mathbf{p}}), \mathbf{h}, \mathbf{u})$ [M A ² /J]	70.62	85.812	21.51 ↑
$^b F_2(\Omega^*(\bar{\mathbf{p}}), \mathbf{h}, \mathbf{u})$ [1/1]	0.11	0.1523	38.45 ↑
$^c F_3(\Omega^*(\bar{\mathbf{p}}), \mathbf{h}, \mathbf{u})$ [M 1/1]	0.898	2.1799	142.75 ↑
$^d F_4(\Omega^*(\bar{\mathbf{p}}), \mathbf{h}, \mathbf{u})$ [10 ⁻³ m]	0.1436	0.0292	79.66 ↓

* worst-case expectation analysis of the third operational mode of the QPR

$$^a F_1(\Omega, \cdot) = \mathbb{E} \left[\frac{1}{2U} \int_{\Omega_S} |\mathbf{h}(\mathbf{p})|^2 d\mathbf{x} \right], \quad ^b F_2(\Omega, \cdot) = \mathbb{E} \left[\frac{\int_{\Omega_S} |\mathbf{h}(\mathbf{p})|^2 d\mathbf{x}}{|\mathbf{h}_{\Omega_S}(\mathbf{p})|^2} \right]$$

$$^c F_3(\Omega, \cdot) = \mathbb{E} \left[\frac{\int_{\Omega_S} |\mathbf{h}(\mathbf{p})|^2 d\mathbf{x}}{\int_{\Omega_F} |\mathbf{h}(\mathbf{p})|^2 d\mathbf{x}} \right], \quad ^d F_4(\Omega, \cdot) = \mathbb{E} \left[\max_{\mathbf{x} \in \Omega_{PS}} (|\mathbf{u}(\mathbf{h})|) \right]$$

ACKNOWLEDGMENTS

This work has been supported by the German Federal Ministry for Research and Education BMBF under contracts 05H21HRRB1, and 05H24HRA.

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