

Modeling of Streamer in Gas by FEM and Semi-Lagrangian Method

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Determining the likelihood of occurrence, growth and extinction of partial discharges between two conductors is a key point when designing electrical devices such as transformers, electrical machines or busbars. The continuity equations governing both time- and position-dependence of charged particle densities enables to build the most comprehensive model of partial discharge development in gas. Known in the literature as fluid model, plasma model or even electrohydrodynamic model, it consists of set of drift-diffusion-reaction type equations strongly coupled to Poisson's equation that still remains challenging to solve by means of FEM due to numerical instability. This paper deals with the modeling of spatio-temporal growth of partial discharge inside gas solved by means of FEM and stabilized by means of a semi-Lagrangian method.

Index Terms—Finite element analysis, partial discharges, conservation equations, Poisson's equation, streamer.

I. INTRODUCTION

IN the past decades, power electronics allowed to drastically increase the efficiency of electrical devices but also led to both higher operating voltage and switching frequency which are responsible for partial discharges (PD) appearance in the electrical insulation system of electrical devices. In order to avoid early aging and thus failure of the latter, the study of PD occurrence in electrical devices recently received considerable attention from designers and researchers. While the ignition, development and extinction mechanism of PD, especially of streamer type, was established in the first half of the previous century [1], [2], efforts in the field of electrical devices focused so far on the determination of the partial discharge inception voltage (PDIV) by means of several empirical criteria such as Paschen's law, Schumann's criterion and Raether-Meek's criterion to name a few [3]. Even though those criteria have shown a certain level of usefulness [4], they are limited when it comes to deeply understand the phenomena involved in partial discharge development.

On the other hand, the continuity equations govern both time- and position-dependence of charged particle densities. When coupled to Poisson's equation, they allow to build a comprehensive model of partial discharge development in gas.

First introduced in the 1960s by Davies [5], this approach is referred to as fluid model, plasma model or even electrohydrodynamic model in the literature and is widely used for plasma numerical modeling [6]. Historically, the finite difference method (FDM) [7] and then the finite volume method (FVM) [8] were used to solve the continuity equations numerically. In the 2000s, Georgiou [9] started to use the FEM to overcome the difficulties encountered by FDM and FVM to deal with complex geometries. Unfortunately, the FEM suffers from numerical instability when it comes to solve drift-diffusion-reaction type equations and requires the use of stabilization methods. Recently, Liu introduced the so-called position-state separation (POSS) method [10] to overcome those numerical instabilities. However, only the diffusive term of the continuity equations were solved by means of FEM

in [10] whereas both the ionization the convective acceleration source terms were solved by means of three-order Runge–Kutta method. This paper aims at solving the whole set of equations, i.e. continuity equations and Poisson's equation, by means of FEM and to derive the appropriate variational formulation which has not been proposed in [10].

II. THEORETICAL BACKGROUND

A. Fluid Model

The fluid model is expressed as [9]:

$$\partial_t n_e = S + (\alpha - \eta) n_e \mu_e |\mathbf{E}| - n_e n_p \beta_{ep} - \nabla \cdot (n_e \mu_e \mathbf{E} - D_e \nabla n_e) \quad (1a)$$

$$\partial_t n_p = S + n_e \alpha \mu_e |\mathbf{E}| - n_e n_p \beta_{ep} - n_n n_p \beta_{np} - \nabla \cdot (n_p \mu_p \mathbf{E}) \quad (1b)$$

$$\partial_t n_n = n_e \eta \mu_e |\mathbf{E}| - n_n n_p \beta_{np} - \nabla \cdot (n_n \mu_n \mathbf{E}) \quad (1c)$$

$$\nabla \cdot (\epsilon \mathbf{E}) = e (n_p - n_n - n_e) \quad (1d)$$

where t is the time, ϵ is the electric permittivity, $\mathbf{E}$ is the electric field, e is the elementary charge, S represents any source/sink mechanism, n_e , μ_e , n_p , μ_p , n_n and μ_n are the charge densities and the mobility for electron, positive and negative ions respectively and D_e is the electron diffusion coefficient. The ionization, attachment, electron–positive-ion recombination and negative-ion–positive-ion recombination coefficients are denoted α , η , β_{ep} and β_{np} .

B. POSS method

The generic form of a transport equation is

$$\partial_t \rho + \nabla \cdot (\mathbf{u} \rho - D \nabla \rho) = s \quad (2)$$

where ρ is the transported quantity, $\mathbf{u}$ is the velocity field, D is the diffusion coefficient and s is the source/sink function. Relations (1a)-(1b)-(1c) can all be put under the form of (2). The convective term $\nabla \cdot (\mathbf{u} \rho)$ is the critical part of this study. The POSS method splits (2) into two sub-problems: state (3) and position (4) and enables to solve each sub-problem with an appropriate method [10].

1) The state sub-problem

The state sub-problem deals with the variation of quantity ρ due to diffusive, convective acceleration and source terms:

$$\partial_t \rho = D \Delta \rho - \rho \nabla \cdot \mathbf{u} + s. \quad (3)$$

Relation (3) can fully or partially be solved by means of FEM on a so-called reference mesh, and the obtained solution is used as the initial condition to solve the position sub-problem (4) during each time step [10].

2) The position sub-problem

The position sub-problem determines the transport of ρ considering the linear convection only:

$$\partial_t \rho + \mathbf{u} \cdot \nabla \rho = 0. \quad (4)$$

The method of characteristics, applied to (4), enables to compute the transport of quantity ρ by integrating $\partial_t \mathbf{x} = \mathbf{u}$. Thus, the convective displacement $\mathbf{x}$ of each node of the reference mesh can be computed. It defines a so-called auxiliary transported mesh. The next step is to project the transported ρ from the auxiliary mesh back to the reference mesh. The consecutive solving of each state and position sub-problems is repeated for each time step.

3) Variational Formulation of the state sub-problem

In (3), the classical convective term $\nabla \cdot (\mathbf{u} \rho)$ from (2) has been replaced by an unusual convective acceleration term $\rho \nabla \cdot \mathbf{u}$. As the weak form of (3) is not found in the scientific literature, the main scientific contribution of this communication is to develop the variational formulation of the state sub-problem. Thus, a Sobolev space H_0^1 with zero values on the boundary is considered. The weak formulation is derived by multiplying the governing equation (3) by a test function $w \in H_0^1(\Omega)$ and integrating over the domain Ω :

$$\int_{\Omega} (\partial_t \rho - D \Delta \rho + \rho \nabla \cdot \mathbf{u} - s) w d\mathbf{x} = 0, \quad \forall w \in H_0^1. \quad (5)$$

Then, applying Green's formula to the diffusive term:

$$\begin{aligned} \int_{\Omega} (w \partial_t \rho + \nabla w \cdot (D \nabla \rho) + w \rho \nabla \cdot \mathbf{u} - w s) d\mathbf{x} \\ - \int_{\partial \Omega} w (D \nabla \rho) \cdot \mathbf{n} ds = 0, \quad \forall w \in H_0^1. \end{aligned} \quad (6)$$

The boundary term vanishes since $w = 0$ on $\partial \Omega$. Using the scalar product operator $(\cdot, \cdot)$ corresponding to $H_0^1(\Omega)$, the variational formulation problem become:

$$(\partial_t \rho, w) + (D \nabla \rho, \nabla w) + (\rho \nabla \cdot \mathbf{u}, w) - (s, w) = 0. \quad (7)$$

The formulation (7) highlights the specificity of this problem with $\nabla \cdot \mathbf{u}$. Indeed, it can be observed from (1a)-(1b)-(1c) that this term contains the strong non-linear coupling with $\mathbf{E}$ and its derivatives.

III. APPLICATION TEST AND RESULTS

The application test is taken from [6]. It consists in an axisymmetric domain of radius $r_{\max}$ and height $z_{\max}$ both equal to 12.5 mm. A vertical electrical field is applied by setting the electric potential v such that $v(z = 0 \text{ mm}) = 0 \text{ V}$ and $v(z = z_{\max}) = 18 \text{ kV}$. The initial conditions are such

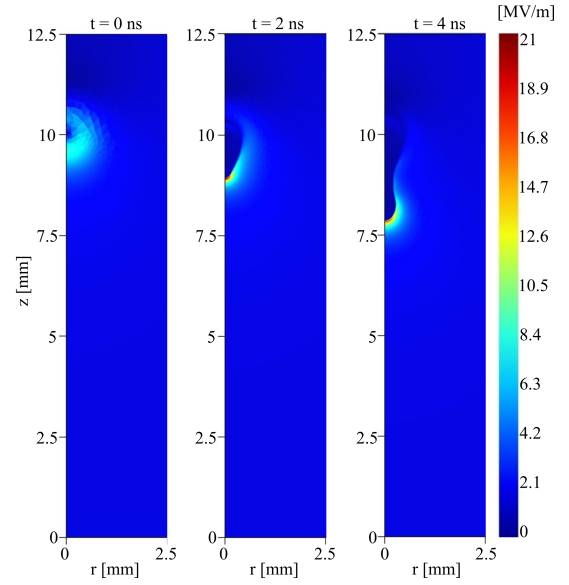


Fig. 1. Electric field strength $|\mathbf{E}|$ during the propagation of the streamer inside the studied domain for $t = 0 \text{ ns}$ (left), $t = 2 \text{ ns}$ (center) and $t = 4 \text{ ns}$ (right). The domain shown here is reduced to $r \in [0, 2.5 \text{ mm}]$ for the sake of readability.

that $n_e = 1 \times 10^{13} \text{ m}^{-3}$ and n_p has a Gaussian distribution of maximum value equal to $5 \times 10^{18} \text{ m}^{-3}$ at $r = 0$ and $z = 10 \text{ mm}$. The preliminary results obtained by using the method presented in Section II-B are shown in Fig. 1 and are consistent with the ones presented in [6]. Details about the weak-formulation as well as as strong coupling processing between electric field and charged particle densities will be developed in the extended paper.

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