

Dynamic simulation of Movement up to Contact Using 2D Coupled Analytical-Finite Element Model

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The present work presents a dynamic simulation of movement up to contact of an electromechanical armature by coupling between an analytical solution (AN) and finite elements (FE) one. The transient behavior of the electromechanical system study with small air-gap is performed with the coupling between mechanical and electromagnetic equations. The completely closed position at the air-gap is investigated with the movement simulation realized using coupled model. In this case only one mesh is needed for all displacements steps. The efficiency obtained when the air-gap vanishes is with a great interest. Results of 2D electromechanical system study using coupled FE-AN model are compared to finite elements calculation. The results seem to be acceptable and promising.

Index Terms—Two-dimensional analytical solution, coupled model, electromechanical system, finite elements, movement simulation.

I. INTRODUCTION

THE Modeling of transient dynamic behavior of electromechanical systems with the presence of small air-gaps is with some difficulties especially when the movement simulation induced at the air-gap necessitates a completely closed position. In numerical modeling this problem is addressed using special methods [1]-[2]-[3]. Also both the use of finite element- boundary element (FE-BEM) and re-meshing techniques need some attention to the singular kernels integration when the boundaries are much closed and increasing number of nodes respectively [2]. The method presented here uses only one mesh for a fixed number of nodes. The association between a finite elements (FE) and analytical solutions (FE-AN) induces separation meshes of static and mobile parts (Fig.2). The movement is then simulated by position changes, obtained from mechanical equation, of nodes at the frontiers of the analytical zone. A coupled FE-AN model yield to less unknown's number and times step with a real up contact than the use of re-meshing technique.

II. ELECTROMECHANICAL MODEL

A. 2D Electromagnetic equation

The electromagnetic phenomena are governed by partials differentials equations of parabolic (transient behavior) or elliptic (static phenomenon) types. These partial differentials equations are obtained from Maxwell's equations by combining constitutive relations and Ohm's law. In the current study we consider a transient 2D electromagnetic equation. The development, in the Cartesian coordinates x and y , yields to the following expression:

$$\frac{\partial}{\partial x} \left(\nu \frac{\partial A_z(x, y)}{\partial x} \right) + \frac{\partial}{\partial y} \left(\nu \frac{\partial A_z(x, y)}{\partial y} \right) - \sigma \frac{\partial A_z(x, y)}{\partial t} = J_{sz} \quad (1)$$

The magnetic vector potential $\vec{A}(T, m)$ and the source current density $\vec{J}_s$ (A/m²) are considered having only one component along z direction: $\vec{A} = (0, 0, A_z)$ and $\vec{J}_s = (0, 0, J_{sz})$ respectively.

ν is the magnetic reluctivity (H/m)⁻¹, σ is the electric conductivity (S/m).

B. Mechanical problem modeling

The displacement of the armature of electromechanical system, induced by the magnetic strength, is assumed to be governed by the classical Newton's equation given below:

$$m \frac{dV(t)}{dt} = F_m \pm F_g \quad (2)$$

m is the mass of the moving object (kg), $V(t)$ represents its velocity (m/s). F_m and F_g are the magnetic and the gravitational forces respectively.

The magnetic strength F_m is obtained from the magnetic co-energy variation with time and expressed by above formula:

$$F_m = - \frac{\partial W_{m-co}}{\partial t} \quad (3)$$

W_{m-co} is the magnetic co-energy of the mobile part (J).

The gravitation strength F_g is computed using (4):

$$W_{m-co} = \iiint_{\Omega} \left(\int_B H \cdot dB \right) d\Omega \quad (4)$$

$$F_g = m g \quad (5)$$

$\vec{B}$ and $\vec{H}$ are the magnetic flux density (T) and the magnetic field (A/m) respectively.

g is the gravitation acceleration (m/s²).

C. Velocity and displacement equations

The expression for the speed is obtained using Euler method for the discretization of (2) as following:

$$\frac{dV(t)}{dt} = \frac{V(t+\Delta t) - V(t)}{\Delta t} \quad (6)$$

$$V(t + \Delta t) = \frac{(F_m \pm F_g)}{m} \Delta t + V(t) \quad (7)$$

Then, the displacement $y(t + \Delta t)$ at time $(t + \Delta t)$ is deduced:

$$y(t + \Delta t) = \frac{1}{2} \frac{(F_m \pm F_g)}{m} (\Delta t)^2 + V(t) \Delta t + x(t) \quad (8)$$

The displacement $y(t + \Delta t)$ is computed at each time step according to the magnetic strength F_m obtained from the resolution of (1) at each time step.

D. Movement simulation technique

The simulation of the movement of electromechanical system is performed by using a coupling between 2D analytical solution in the air-gap and a finite elements resolution (FE-AN) in the others parts of the solving domain. The use of analytical solution coupled to finite elements permits to avoid meshing the air-gap region and consequently re-meshing domain is not required.

The analytical solution of Laplace equation in the air region between yoke and armature of the electromechanical system is given by [4]:

$$A^{AN}(x, y) = \sum_{i=1}^{nt} \left(\sum_{n=0}^{\infty} B_{ni} \frac{\sinh(\frac{n\pi}{b} y_2) - \tanh(\frac{n\pi}{b} y_1) \cosh(\frac{n\pi}{b} y_2)}{\sinh(\frac{n\pi}{b} y_1) - \tanh(\frac{n\pi}{b} y_2) \cosh(\frac{n\pi}{b} y_1)} \sin(\frac{n\pi}{b} x) \right) A_i \quad (9)$$

a and b are the width and the length of the analytical region, y_1 and y_2 are the coordinates of the lower and upper boundaries of the analytical region.

B_{ni} are coupling coefficients [4]. nt is the total number of nodes at both frontiers y_1 and y_2 of the analytical region.

When the air-gap width vanishes ($y_2 = y_1$), the analytical solution (9) is reduced to:

$$A^{AN}(x, y) \Big|_{y_2 \rightarrow y_1} = \sum_{i=1}^{nt} \left(\sum_{n=0}^{\infty} -B_{ni} \sin(\frac{n\pi}{b} x) \right) A_i \quad (10)$$

The analytical solution (10) gives finite values when the air gap width vanishes. This is an important result for modeling very small or a perfect contact between objects.

III. APPLICATION AND RESULTS

The transient analysis of the actuator study consists to move the armature up to contact in the opposite direction of oy [2].

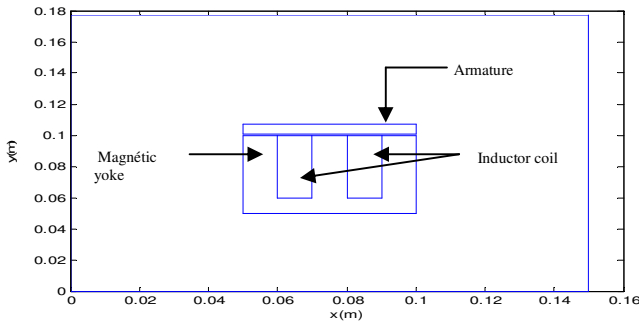


Fig. 1. Geometry of Electromechanical system

In Fig. 1 is presented the studied geometry of the electromechanical device. After resolution of the coupled model we present the 2D distribution of the magnetic vector potential as shown if Fig. 2. This result show that the 2D analytical solution in the air gap gives a good representation of the electromagnetic phenomenon and the coupling with the finite elements solution is with a good quality. Then the position changes of the armature are computed at each

displacement step taking into account the interactions between the electromagnetic and mechanical phenomena. Position changes of the frontier nodes at the top air-gap permits to simulate the movement using only one mesh.

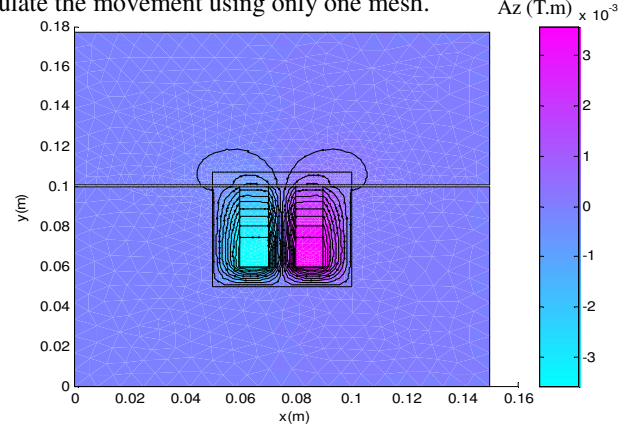


Fig.2. Magnetic vector potential distribution of coupled model

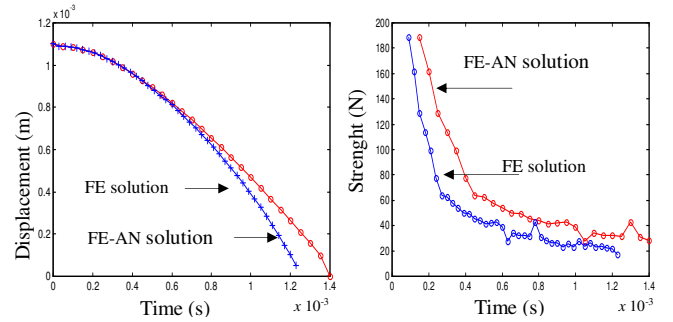


Fig.3. Comparisons between coupled FE-AN and finite elements models

- (a) : Displacement behavior with time
(b) : Strength behavior with time

The results obtained using coupled FE-AN model show that the air gap width decreases until zero value (which is a theoretical value) (Fig. 3.a). But in case of finite elements model the mesh of the air gap is limited to a non-zero critical value of the air gap width. A computational time is also reduced as only one mesh is used for all displacements steps comparing to finite elements resolution. Confrontation between results of coupled (FE-AN) and finite elements (FE) models show a good agreement for both strength and displacement behaviors (Fig.3.a, Fig.3.b).

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