

Numerical Simulation of Magnetic Field Effects on Symmetrically Collapsing Fluids

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We present numerical simulations of magnetic field effects in symmetrically collapsing fluids. Our Python-based Lagrangian solver couples compressible fluid dynamics with magnetic field evolution on a non-uniform radial mesh, advancing in time via an implicit Newton–Raphson integrator and employing artificial viscosity for robust shock capturing. Implosion diagrams reveal that a toroidal magnetic field can accelerate the collapse and trigger a small-scale instability. This digest distills the numerical method and key results to demonstrate the solver performance on one hand and to introduce remarkable aspects of MHD collapse dynamics on the other.

Index Terms—Fluid collapse, Magnetohydrodynamics (MHD), Navier-Stokes equations, Numerical methods, Python simulation.

I. INTRODUCTION

THE numerical computation of dynamical phenomena in plasmas, such as shock waves during implosion or collapse, relies on coupling compressible fluid dynamics with electromagnetic fields. While many contemporary codes target complex 3D geometries—e.g. tokamaks or stellarators—much of the fundamental physics can already be elucidated in a 1D, spherically symmetric setting [1]. Reduced models demand rigorous error control to separate genuine physical effects from numerical artifacts [2].

Although (1)–(3) are written in the standard *Eulerian* form, our implementation discretizes them in a fully *Lagrangian* framework, moving the radial grid with the fluid shells. Furthermore, in our model the toroidal magnetic field B is everywhere perpendicular to the inward radial velocity u , similar to hoops tightening around a collapsing sphere, so that the resulting Lorentz force acts purely in the radial direction.

In this work, we solve the following MHD equations:

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r}(r^2 \rho u) = 0, \quad (1)$$

$$\rho \frac{Du}{Dt} = -\frac{\partial p}{\partial r} + \underbrace{\frac{1}{\mu_0 r} \frac{\partial}{\partial r}(r B^2)}_{\text{Lorentz force}} + Q \quad (2)$$

$$+ \underbrace{\nu \left(\frac{\partial^2 u}{\partial r^2} + \frac{2}{r} \frac{\partial u}{\partial r} - \frac{2u}{r^2} \right)}_{\text{viscous term}},$$

$$\frac{\partial B}{\partial t} = -\underbrace{\frac{1}{r} \frac{\partial}{\partial r}(r u B)}_{\text{advection}} + \underbrace{\eta \left(\frac{\partial^2 B}{\partial r^2} + \frac{2}{r} \frac{\partial B}{\partial r} \right)}_{\text{resistive diffusion}}, \quad (3)$$

$$p = K \rho^\gamma, \quad \gamma = \frac{5}{3}. \quad (4)$$

Notation: $\rho(r, t)$ is the mass density (kg/m³), $u(r, t)$ the radial velocity (m/s), p the gas pressure (Pa), and $B(r, t)$ the toroidal magnetic field (T). The pressure scale is ensured by the polytropic constant K (m⁴ · kg^{-2/3} · s⁻²). The operator $\frac{D}{Dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial r}$ is the *material derivative*, tracking the rate of change experienced by a fluid element. Q denotes Von

Neumann–Richtmyer artificial viscosity for shock smoothing, ν the kinematic viscosity (m²/s), η the magnetic diffusivity (m²/s), and μ_0 the vacuum permeability (H/m).

To trigger the implosion, we add an external forcing term $F(r, t)$ in the momentum equation (2), representing a Gaussian pulse localized near the outer boundary with peak speed $u_{\max} = 1.4 c_s$, where $c_s = \sqrt{\gamma K \rho_0^{\gamma-1}}$ is the reference sound speed.

Capturing shocks, Lorentz forces, and diffusion demands exact mass conservation and stable magnetosonic handling. We employ a compact Python Lagrangian solver with implicit backward-Euler/Newton–Raphson–Krylov time stepping on a clustered grid and a tridiagonal magnetic-field update. Implosion diagrams then visually confirm solver accuracy and emerging MHD dynamics.

II. NUMERICAL METHOD

Our one-dimensional, spherically symmetric MHD solver calculates four primary fields on a Lagrangian mesh: the radial node positions r_i , the fluid velocity u_i , the toroidal magnetic field B_i , and the fixed shell masses m_i . At each time step, the following sequence is applied:

- 1) **Non-uniform Lagrangian grid.** We generate $N + 1$ radial nodes by

$$r_i = r_{\min} + (r_{\max} - r_{\min})[1 - (1 - i/N)^\alpha], \quad \alpha = 3,$$

clustering resolution near the outer boundary to capture precisely the stress-free boundary and nascent shock structure [3]. Shell masses are computed once and remain fixed, enforcing exact mass conservation.

- 2) **Momentum update.** Cell-centered pressure is computed from the polytropic law (4). Nodal accelerations are assembled from

- hydrostatic pressure gradients,
- Von Neumann–Richtmyer artificial viscosity for shock smoothing [4],
- Lorentz force due to the toroidal field,
- physical (Navier–Stokes) viscosity.

- 3) **Implicit time integrator.** After discretizing spatial derivatives via second-order finite differences, the PDEs reduce to a stiff system of ODEs in time. We advance them with the implicit backward-Euler method, solving each nonlinear algebraic problem by Newton–Raphson iteration [5]. The linearized systems are tackled with a Jacobian-free GMRES solver [6].
- 4) **Magnetic-field update.** We advance the induction equation (3) by operator splitting. First, the toroidal field is advected explicitly along the updated Lagrangian velocity field using the same nodal connectivity (this remains stable since the grid moves with the flow). Next, we apply the resistive diffusion step implicitly: a backward-Euler discretization of the diffusion term yields a tridiagonal linear system, which we solve efficiently with the Thomas algorithm [7]. This two-stage approach cleanly separates advection and diffusion and ensures $\nabla \cdot \mathbf{B} = 0$ to hardware precision.
- 5) **Snapshot & verify.** At each step we record ρ_i , u_i , and B_i trajectories to build implosion diagrams. Because shell masses are fixed and each node’s motion is computed exactly, mass conservation is enforced by construction, and the smoothness of shell trajectories offers an immediate visual check of numerical accuracy.

III. SIMULATION SETUP AND IMPLOSION DIAGRAMS

A. Simulation Setup

All cases use the same solver and parameters; only the initial toroidal field B_0 varies (0 or 1.35 T).

- **Domain & grid.** Spherical shell $r \in [0.1, 0.6]$ m with $N = 442$ Lagrangian cells.
- **Initial state.** Uniform density $\rho = 1$ kg/m³, zero velocity $u = 0$, and $B(r, 0) = B_0$.
- **Forcing pulse.** Gaussian velocity pulse on the outer 5% of shells, peak $1.4 c_s$, centered at 0.15 ms with ± 0.075 ms decay.
- **Time stepping & parameters.** Implicit backward-Euler with $\Delta t = 10^{-6}$ s up to $t_{\max} = 1.5 \times 10^{-3}$ s; $\nu = 10^{-3}$ m²/s, $\eta = 5 \times 10^{-2}$ m²/s.
- **Boundary conditions.** Inner $r = 0.1$ m (to avoid a singularity) fixed ($u = 0$); outer $r = 0.6$ m enforces zero radial gradients on p and B .

B. Early-Time Implosion Diagrams

An *implosion diagram* overlays the radial trajectories $r_i(t)$ of all Lagrangian shells. The shock front appears as the envelop of maximum convergence, while any fine-scale ripples or instabilities become immediately apparent [3].

IV. CONCLUSION

We present a compact 1D Lagrangian MHD solver that precisely conserves mass, captures shocks, and integrates both hydrodynamic and magnetic dynamics. The early-time implosion diagrams show that a toroidal field B markedly accelerates the collapse and imprints a fine-scale structure on the shock front, underlining the critical role of the magnetic

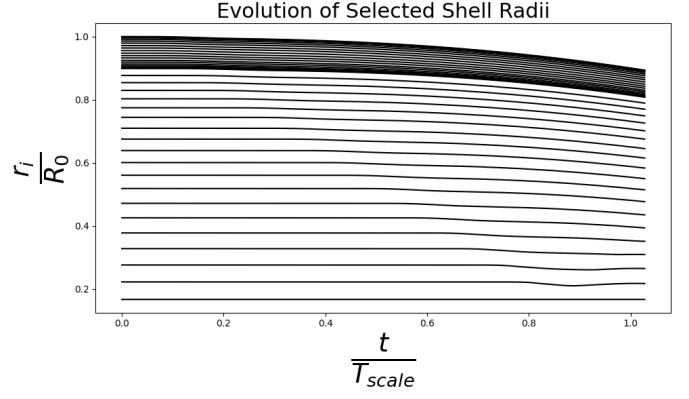


Fig. 1: Pure hydrodynamic implosion ($B_0 = 0$) under a Mach 1.4 pulse. Shells accelerate inward smoothly, forming a well-defined shock front at early times. $R_0 = 0.6$ m and $T_{\text{scale}} = 1$ ms.

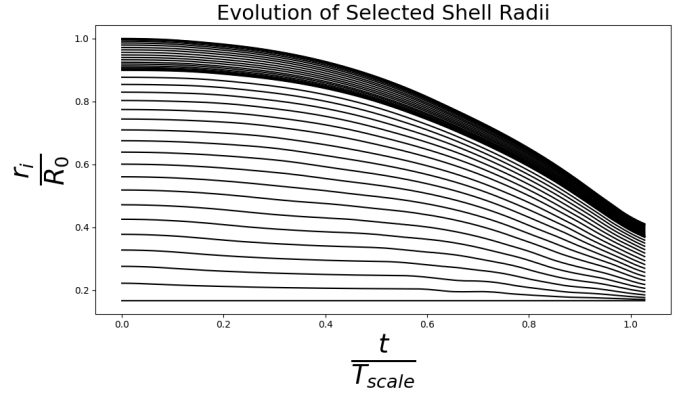


Fig. 2: MHD implosion ($B_0 = 1.35$ T) under the same Mach 1.4 pulse. The magnetic field accelerates inner shells and imprints fine-scale ripples on the nascent shock front. $R_0 = 0.6$ m and $T_{\text{scale}} = 1$ ms.

field. This user-friendly toolkit can be extended to longer times and broader parameter scans to fully characterize emerging MHD instabilities. First attempts showed interesting qualitative changes in the shock wave related to the magnetic Reynolds number of the system, which is currently systematically investigated.

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