

Adaptive Time-stepping Model Order Reduction Approach for Nonlinear System

Sance Gao, Fan Yang and Cheng Chi

State Key Laboratory of Power Transmission Equipment and System Security and New Technology, 400044 Chongqing, China

Proper orthogonal decomposition is a widely used method for model order reduction, offering an efficient means to simplify complex systems. However, it has certain limitations, particularly in handling nonlinear problems and its reliance on fixed time-step computations in the time domain, which can be computationally inefficient. This paper presents an enhanced model order reduction approach based on proper orthogonal decomposition, incorporating adaptive time-stepping to address these challenges. The proposed method corrects the time step size using truncation error; the degree of nonlinearity is considered in updating step size to improve convergence. To validate the approach, a converter transformer is used as a case study, demonstrating the effectiveness of the adaptive time-stepping model in evaluating the insulation state under polarity reversal conditions.

Index Terms—Adaptive time-stepping, model order reduction, nonlinear system, proper orthogonal decomposition

I. INTRODUCTION

THE polarity reversal of converter transformers, which occurs within a short time, poses a significant insulation hazard, potentially compromising operational safety. Accurate estimation of the insulation state during the design stage is crucial but presents several challenges. Firstly, the large size of converter transformers and the presence of intricate components result in a computational burden [1]. Secondly, polarity reversal tests focus on rapid electric field variations during the reversal process [2], while much of the test duration involves relatively stable conditions. This makes fixed time-step simulations inefficient and uneconomical.

To address these challenges, this paper proposes an adaptive time-stepping model order reduction approach. The method employs proper orthogonal decomposition to construct a reduced-order model and adaptively adjusts the time step size by accounting for the degree of nonlinearity.

II. EQS PROBLEM IN POLARITY REVERSAL

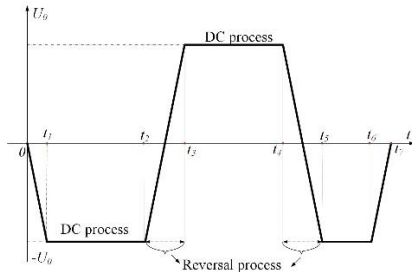


Fig. 1. Voltage curve in polarity reversal test.

Converter transformers can experience polarity reversal conditions, characterized by rapid changes in voltage polarity within a few seconds, as depicted in Fig. 1. This voltage excitation can be considered a low-frequency phenomenon. In oil-paper insulation systems, which exhibit low conductivity, conduction current densities and displacement current densities have comparable magnitudes. Furthermore, the influence of magnetic field variations on the electric field strength is negligible. Consequently, the insulation behavior can be

accurately modeled using the electro-quasi-static (EQS) field approximation, as described by the following equation

$$\begin{cases} \nabla \cdot \left(\epsilon \frac{d}{dt} + \gamma \right) \nabla \varphi = 0 & \text{in } \Omega \\ \varphi = u(t) & \text{on } \Gamma_1 \\ \frac{d\varphi}{dn} = 0 & \text{on } \Gamma_2 \end{cases}, \quad (1)$$

where φ is the electric potential; ϵ is the permittivity and σ is the nonlinear conductivity depending on electric field.

After applying time and spatial discretization using the finite element method, the governing equation is formulated as:

$$(K_\epsilon + \Delta t K_\gamma(\varphi)) \varphi_{m+1} = K_\epsilon \varphi_m \quad (2)$$

where K_ϵ is the permittivity matrix, K_γ is the conductivity matrix, and m is the m -th time step.

III. ADAPTIVE TIME-STEPPING MODEL ORDER REDUCTION

A. Proper Orthogonal Decomposition

The proper orthogonal decomposition method is employed to develop this reduced-order model for EQS problem. Based on few samples, the orthogonal basis would be generated by singular value decomposition [3], which would provide reduced-order subspace to solve unknown variable:

$$A = P \begin{bmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \ddots \end{bmatrix} V^H \quad (3)$$

where P is a sample matrix containing time-domain snapshots, and V is an orthonormal matrix. $\text{Diag}(\lambda_1, \lambda_2, \dots)$ contains Nm singular values capturing the most energy of the system, usually $Nm \ll Ns$ (Ns is the degree of freedom of this system).

By the orthogonal basis, the unknown variable can be approximated by

$$\varphi_m = P \alpha_m \quad (4)$$

where α_m is an unknown variable after order reduction.

Substituting (4) into (2) and multiplying the matrix P^T , the governing equation in reduced order way is shown as:

$$P^T (K_\epsilon + \Delta t K_\gamma(\varphi)) P \alpha_{m+1} = P^T K_\epsilon P \alpha_m \quad (5)$$

It is noteworthy that while the unknown variables are computed within the reduced-order subspace, a full-order update is necessary for each time step to account for nonlinearity.

B. Adaptive Time-stepping Scheme

In the polarity reversal test, the DC process typically extends over 90 minutes, while the reversal process occurs within tens of seconds. Besides, the electric field undergoes significant and rapid changes during the reversal process, smaller time steps are needed to accurately capture these dynamics. Conversely, larger time steps can be effectively utilized to observe the DC process. Consequently, an adaptive time-stepping algorithm provides a more economical and accurate approach compared to a fixed step size method for the scenario of polarity reversal.

At the m -th time step, the truncation error R_m is evaluated by [4]

$$R_m = \frac{h_m^2}{12} \frac{\partial^2 \varphi(\zeta)}{\partial t^2}, \quad (6)$$

where h_m is the step-size, namely $h_m = t_m - t_{m-1}$, and $\frac{\partial^2 \varphi(\zeta)}{\partial t^2}$ is solved by previous results. ζ is any constant between t_{m-1} and t_m , namely $\zeta \in (t_{m-1}, t_m)$.

Since $R_m \propto h_m^2$, the truncation error for the $(m+1)$ -th time step, R_{m+1} , will also be proportional to the square of the step size h_{m+1} at the $(m+1)$ -th time step:

$$R_{m+1} = \frac{h_{m+1}^2}{12} \frac{\partial^2 \varphi(\zeta_{m+1})}{\partial t^2} = \frac{h_{m+1}^2}{12} \frac{\partial^2 \varphi(\zeta_m)}{\partial t^2} = \frac{h_{m+1}^2}{h_m^2} R_m. \quad (7)$$

To ensure that the truncation error at the $(m+1)$ -th time step, R_{m+1} remains within the specified tolerance ($R_{m+1} < R_c$). The following approach can be adopted.

$$h_{m+1} \leq h_m \sqrt{\frac{R_c}{|R_m|}}. \quad (8)$$

Introducing a safety coefficient $K_{sf} \in [0.8, 0.9]$ yields the following:

$$h_{m+1} = K_{sf} h_m \sqrt{\frac{R_c}{|R_m|}}. \quad (9)$$

The moment of the $(m+1)$ -th time step is determined as follows: $t_{m+1} = h_{m+1} + t_m$.

C. Step Size in Nonlinear System

In nonlinear systems, it is crucial to consider the degree of nonlinearity when determining the time step size, which impacts the convergence rate of the computation [5]. Consequently, a correction coefficient involving the nonlinear degree is introduced.

$$\alpha = \max \alpha_n = \max \frac{\|\varphi_m - \varphi_{m-1}\|}{\|\varphi_{m-1} - \varphi_{m-2}\|} \quad (10)$$

where α is the correction coefficient. If α exceeds 1, it indicates significant variation in the results within the current time step. Consequently, the next time step size should be decreased to improve solution accuracy. Conversely, if α is less than 1, the results exhibit minimal variation, suggesting that the current

time step size can be safely increased to enhance computational efficiency.

By the correction coefficient, the step size is defined as:

$$h_{m+1} = \frac{\alpha_{ref}}{\alpha} K_{sf} h_m \sqrt{\frac{R_c}{|R_m|}}. \quad (11)$$

IV. APPLICATION

A converter transformer model is utilized as a case study to investigate insulation performance under polarity reversal conditions. In this scenario, the valve-side winding is subjected to a polarity reversal, as illustrated in Fig. 1, while the grid-side winding and the oil tank are grounded. Employing an adaptive time-stepping model, the electric potential distribution within the converter transformer is analyzed and presented in Fig. 2. The variations of electric potential along time is depicted in Fig. 3.

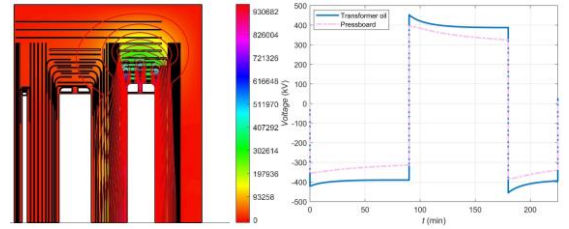


Fig.2. Electric potential distribution. Fig.3. Electric potential along time.

V. CONCLUSION

This paper introduces an adaptive time-stepping approach for model order reduction, specifically tailored to systems exhibiting nonlinear behavior. The proposed method utilizes proper orthogonal decomposition and incorporates a novel strategy for updating the time step size based on the degree of nonlinearity within the system. To demonstrate the efficacy of this approach, a converter transformer is employed as a case study. The implementation and results of this method, including the distribution of electric fields, are presented.

REFERENCES

- [1] C. Chi, F. Yang, and Z. Ren. "Reduced order model based on combined POD/LDEIM-Q for nonlinear thermoelectric coupling." *IEEE Transactions on Magnetics*, vol.58, no.9, 2022.
- [2] T. Terakura, K. Takano, T. Yasuoka, et al. "Transient electric field analysis in consideration of the electric field dependence of resistivity in the converter transformer." *Electrical Engineering in Japan*, vol.198, no.1, pp.3-11, 2017.
- [3] MD. Rokibul Hasan, L. Montier, T. Henneron, et al. "Stabilized reduced-order model of a non-linear eddy current problem by a gappy-POD approach." *IEEE Transactions on Magnetics*, vol.54, no.12, 2018.
- [4] W. N. Fu and L.H. Siu. "Elimination of nonphysical solutions and implementation of adaptive step size algorithm in time-stepping finite-element method for magnetic field-circuit-motion coupled problems." *IEEE transactions on magnetics*, vol.46, no.1, pp.29-38, 2009.
- [5] A. M. P. Valli, G. F. Carey, and A. L. G. A. Coutinho. "Control strategies for timestep selection in simulation of coupled viscous flow and heat transfer." *Communications in Numerical Methods in Engineering*, vol.18, no.2, pp.131-139, 2002.