

Semi-Analytical Algorithm for Electromagnetic Vibration of Racetrack-Shaped Wound Core

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This paper proposes a semi-analytical algorithm to study electromagnetic vibrations in racetrack-shaped wound cores. The algorithm incorporates elliptical approximation for analytical simplifications while accounting for the asymmetric magnetostrictive properties of silicon steel. Compared to finite element methods, this approach achieves faster computation and enhanced accuracy. The study explores deformation states, magnetostrictive performance, and magnetic field distribution within the core. Experimental results using a laser vibrometer validate the displacement calculations, showing consistency with theoretical predictions. This work contributes a novel and efficient computational method for analyzing magnetostrictive vibration phenomena in anode saturable reactors (ASR), a key component in high-voltage direct current (HVDC) systems.

Index Terms—Core vibration, elliptical approximation, magnetostriction, racetrack-shaped core, semi-Analytical Algorithm.

I. INTRODUCTION

The Anode Saturable Reactor (ASR), a key component in HVDC converter valves, limits current rise during thyristor turn-on [1]. The widely used racetrack-shaped wound core design simplifies manufacturing, installation, and winding assembly.

The high operating frequency of the ASR amplifies vibration issues. Current studies address this by using finite element models to calculate parameters like magnetic flux density and displacement via magneto-mechanical coupling [2]. However, simulation convergence heavily depends on boundary conditions and solver selection, while obtaining smooth displacement and acceleration curves requires significant computation time. Additionally, the hysteresis and nonlinear magnetic domain behavior in silicon steel result in a "butterfly-shaped" magnetostrictive curve, with differing trajectories for increasing and decreasing magnetic fields, complicating accurate finite element simulations [3].

This paper presents a semi-analytical method for analyzing electromagnetic vibrations in racetrack-shaped wound cores using an elliptical approximation. The core's elliptical simplification enables analytical calculations while incorporating the asymmetric magnetostrictive properties of silicon steel. This method provides faster computation and greater accuracy compared to the finite element method.

II. RESEARCH FOUNDATION

A. Analysis of Deformation States

The finite element simulation results of the ASR core vibration under an average magnetic flux density of 1 T and an operating frequency of 400 Hz are shown in Fig. 1. Over one excitation cycle T , the core undergoes periodic deformation due to the magnetostrictive effect. At $T/8$, the curved section of the core exhibits slight expansion, and at $T/4$ the deformation

reaches its peak, with significant stretching observed in the curved section.

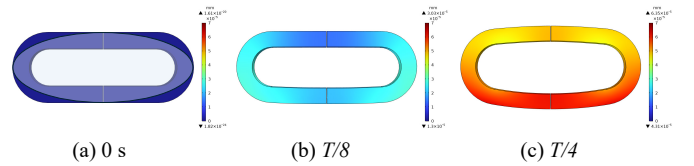


Fig. 1. Finite element calculation results of the ASR core vibration.

The electromagnetic force at the air gap integrates the two U-shaped halves of the core, with deformation concentrated in the curved sections. The core's symmetric shape allows simplification as an ellipse for analytical calculations of deformation. The initial ellipse, shown in Fig. 1(a), aligns its four endpoints with the core's outer contour, enabling quantitative determination of endpoint vibration displacement.

B. Characterization of Magnetostriction

The magnetostrictive performance of a core is typically characterized by the strain under specific magnetic fields. In ASR wound cores, there are three directions: rolling, transverse-to-rolling, and lamination [4]. When simplified to an ellipse, only the rolling direction is considered. Assuming the magnetostrictive strain equals the silicon steel's magnetostrictive coefficient, the core perimeter under a given magnetic flux density is expressed as:

$$l(B) = l_0 [1 + \varepsilon(B)] \quad (1)$$

where l_0 is the original length of the core, B is the magnetic flux density, and $\varepsilon(B)$ is the strain of the core under a given magnetic flux density, determined from measurements of the silicon steel sheets.

C. Semi-Analytical Computational Process

The flowchart of the proposed semi analytical method for electromagnetic vibration is shown in Fig. 2. This semi-analytical method involves iterative calculation of the magnetic flux density $B(t)$ based on the magnetic circuit and μ - B

relationship. The strain $\varepsilon(t)$ is obtained by fitting the magnetostrictive butterfly curve $\varepsilon(B)$ and substituting $B(t)$. Finally, the core's elliptical shape is analyzed to compute the perimeter, short axis, and major axis, enabling the determination of endpoint displacement.

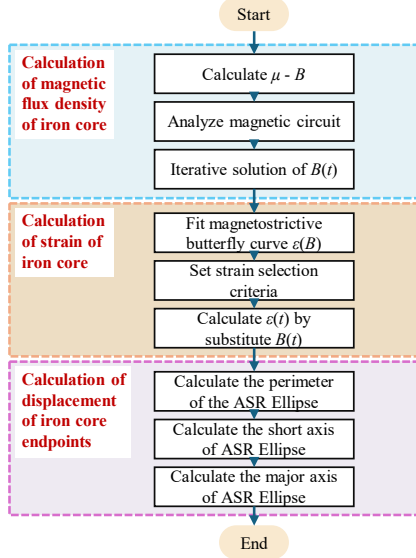


Fig. 2. Flow chart of semi analytical algorithm for electromagnetic vibration.

III. MAGNETIC FIELD CALCULATION

The complete magnetic circuit of the ASR core consists of the core and the air gap. Assuming a uniform magnetic flux density within the core and neglecting the influence of magnetic leakage, the magnetic flux density of the core can be calculated based on the relationship between magnetomotive force and magnetic flux:

$$B = \frac{nI}{\frac{l_g}{\mu_0} + \frac{l_c}{\mu(B)}} \quad (2)$$

where B is the magnetic flux density (T), n is the coil turns, I is the current (A), l_g and l_c are the air gap and core lengths (m), μ_0 is the vacuum permeability, and $\mu(B)$ is the core permeability dependent on B .

IV. STRAIN AND DISPLACEMENT CALCULATION

The magnetostrictive butterfly curve is divided into two parts, “-x T~x T” and “x T~-x T” ensuring that the strain trend in each part matches the measured data. Due to the symmetry of the butterfly curve, only one part is fitted using a high-order polynomial. In this study, a butterfly curve with a magnetic flux density peak of 1 T is used as an example. It should be noted that x represents the peak magnetic flux density of the core. For computational convenience, the slopes at the tail ends of Curve 1 and Curve 2 are constrained to sum to zero during the fitting process. The fitting results are shown in Fig. 3.

Given $B(t)$ and $\varepsilon(B)$, $\varepsilon(t)$ can be derived. From (1), $\varepsilon(t)$ determines the ellipse's perimeter. The minor axis is proportional to the ASR core's curved section, which also satisfies (1). Using Ramanujan's formula for the perimeter, the major axis length is solved from the following equations:

$$\begin{cases} b(t) = b_0 * [1 + \varepsilon(t)] \\ c(t) = c_0 * [1 + \varepsilon(t)] \\ c(t) = \pi \{ 3[a(t) + b(t)] - \sqrt{[3a(t) + b(t)] * [a(t) + 3b(t)]} \} \end{cases} \quad (2)$$

where a , b , and c refer to the semi-major axis, semi-minor axis, and perimeter of the ellipse. The time-varying displacement of the core's major axis endpoints is shown in Fig. 4.

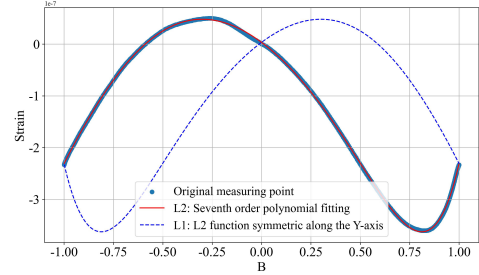


Fig. 3. Fitting Results of Magnetostrictive Curve.

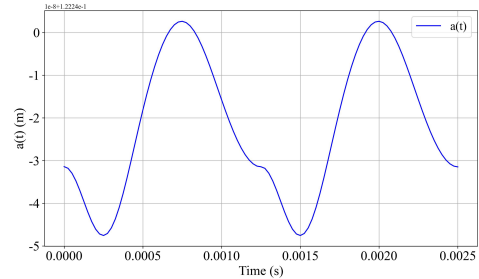


Fig. 4. Displacement results at the endpoints of the core's major axis.

V. EXPERIMENT

The experimental setup is shown in Fig. 5. The ASR core endpoint displacement was measured with a laser vibrometer, aligning with Fig. 4 and showing asymmetry between half-cycles. Additional data will be provided in the manuscript.



Fig. 5. Experiment apparatus.

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