

Equivalent Model for Mechanical Motion Based on Extended Cauer Ladder Circuit

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This paper presents an equivalent model for mechanical motions based on Cauer ladder circuit. It is important for electro-mechanical coupling systems to evaluate their performance as well as their control systems. For such cases, small-scaled equivalent models so called 1-D models are often used. When constructing 1-D models, if mechanical model can be constructed as circuit model, one can integrally evaluate all the systems including control systems. For this reason, mechanical motion models are constructed using an extended Cauer ladder circuit model.

Index Terms—Coupled analysis, Equivalent circuit model,

I. INTRODUCTION

ELECTRO-MECHANICAL coupling systems are one of the key and common components of power and infrastructure systems, such as motors, generators, and so on. In order to develop such systems, their performance should be evaluated considering electro-mechanical coupling: e.g. vibration analysis due to magnetic force. There are also some cases in which strong electro-mechanical coupling must be considered. One of such applications is energy harvesting, especially vibration energy harvesters [1]. Moreover, as for the energy harvesting systems, the circuit and control systems are also very important. For this reason, electro-mechanical, circuit, and control systems must be considered to correctly evaluate their performance. In such complex coupled analysis, small-scaled equivalent models so called 1-D models are often used. However, to integrally consider multiple different systems, different 1-D models must be coupled. If mechanical motion can be modeled as circuit, we can utilize well-developed circuit simulation software including their control systems.

In this work, therefore, we introduce an extended Cauer ladder circuit to model mechanical motions. The Cauer ladder circuits have recently been utilized to model the frequency characteristics of electromagnetic systems [2], [3]. Such Cauer circuits are modified for the equation of motion. It is shown that mechanical motions can be replicated using the extended Cauer circuits.

II. EQUIVALENT CAUER LADDER CIRCUIT MODEL FOR MOTION

The equation of motion in frequency domain is given by

$$(j\omega M + C_d)\hat{V}_{el} + K\hat{X} = \hat{F}, \quad (1)$$

where M , C_d , K are mass, damping coefficient, spring coefficient, respectively. In addition, $\hat{F}$ denotes external force and $\hat{X}$ is position, and obviously,

$$\hat{V}_{el} = j\omega\hat{X}. \quad (2)$$

It is well known that the equation of motion can be translated as

$$\left\{ \frac{1}{C_d} + j\omega M + \frac{1}{j\omega K} \right\} \hat{V} = \hat{F}. \quad (3)$$

It is obvious that (3) can be expressed by a circuit model using L , C , and R with a constant current source.

On the basis of the above-mentioned simple equivalent circuit model, we introduce an extended Cauer circuit model to represent mechanical motion, as shown in Fig. 1. The present circuit model consists of pairs of the parallel R - L and R - C circuits. By determining the circuit constants of these R , L , and C , equivalent circuit models for mechanical motions can be constructed.

In this work, we propose two methods to determine the circuit constants. First, a numerical optimization approach is introduced. This approach is effective for a few degrees of freedom model like (1). Note that (1) can be modeled only by one stage ladder circuit systematically, that is, $R_L = R_C = 2/C_d$, $L = 1/K$, $C = M$. However, for example, M and C_d depend on frequency in case of floating objects. In such cases, more than two ladder circuits are effective; such critical application is wave energy conversion systems. Because there is no theory to determine the circuit constants for after the second stages of the ladder, these constants are numerically optimized to fit given characteristics.

The second approach to determine the circuit constants is an iterative algorithm based on Cauer Ladder Network (CLN) method [4]. When finite element models are given, the equation of motion in s domain is written by

$$\begin{bmatrix} -1 & s \\ sM & K \end{bmatrix} \begin{bmatrix} V_{el} \\ X \end{bmatrix} = \begin{bmatrix} 0 \\ f_{ex}(s)f \end{bmatrix}, \quad (4)$$

where M and K are the mass and stiffness matrix of FEM, respectively, and X and V_{el} denote the displacement vectors of

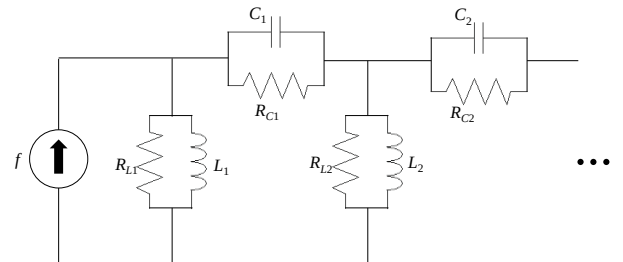


Fig. 1. Extended Cauer circuit to model mechanical motion.

the nodes and their velocity, respectively. By applying CLN-based algorithm to (4), we can obtain the following equations:

$$U_1 + s\lambda_1 V_2 = f_{ex}, \quad (5)$$

$$U_{2n+1} + s\lambda_{2n+1} V_{2n+2} - s\lambda_{2n} V_{2n} = 0, \quad (6)$$

$$s\lambda_{2n+1} U_{2n+1} - s\lambda_{2n-1} U_{2n-1} + V_{2n} = 0. \quad (7)$$

Assuming that $sL_{2n+1} = s\lambda_{2n+1}$ and $sC_{2n+2} = s\lambda_{2n+2}$, (5)-(7) show the circuit equation shown in Fig. 1. Consequently, we can automatically construct an equivalent circuit model for a given FE model for mechanical motion.

III. NUMERICAL EXAMPLE

First, the behavior of a floating object is modeled. In this work, as a floating object model, RM3 [5] is used, as shown in Fig. 2. The frequency characteristics of M and C_d of the float of RM3 are computed using the WAMIT® software [6]. The frequency response of the float and that computed by the present Cauer circuit model are shown in Fig. 3. We can see that the present 3-stages circuit can accurately express the original response, while 1-stage classical circuit cannot perfectly do the response. The time variation of the displacement when sinusoidal force with angular frequency is 2 rad/s is shown in Fig. 3. We can see that almost equivalent results are obtained.

Second, a ring FE model is prepared. Radial force is imposed on it, as shown in Fig. 5. The present iterative algorithm is applied to this model. As a result, a 3-stages circuit model is automatically obtained. The frequency response of this model is shown in Fig. 6, which shows that the present circuit can express the analytical 2nd and 4th diameter node modes.

From these results, we conclude that the present extended Cauer circuit can realize various equivalent models for mechanical motions.

In the fullpaper, the derivation process to determine the circuit constants will be mentioned in detail, and more than two degree of freedom model will be introduced.

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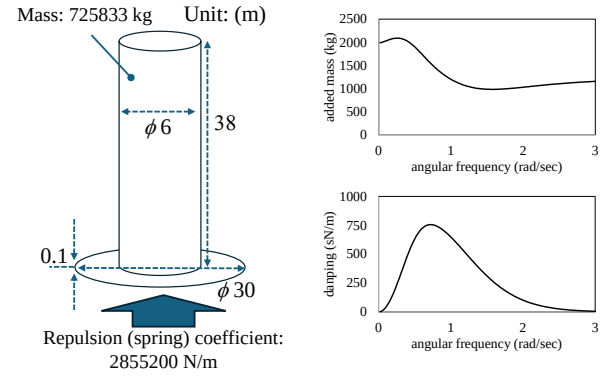


Fig. 2. Floating object model called RM3.

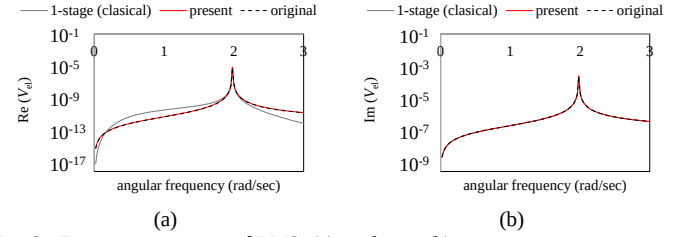


Fig. 3. Frequency response of RM3, (a): real part, (b): imaginary part.

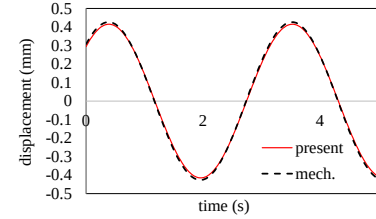


Fig. 4. Time variation of displacement when $\omega=2\text{rad/s}$.

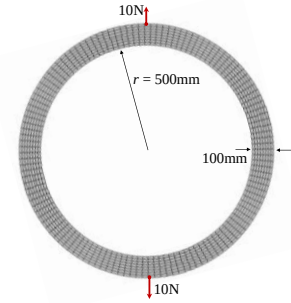


Fig. 5. Ring finite element model.

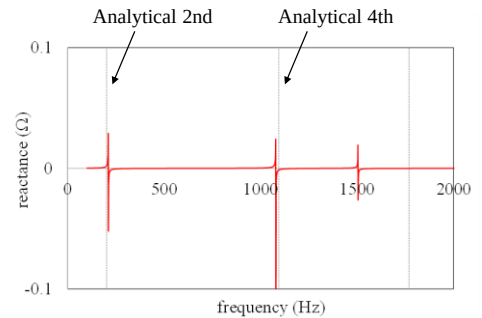


Fig. 6. Frequency response of ring model.