

Physics-driven Electro-magnetic Particle Coupling Calculation Method for Vacuum Electron Devices

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Trajectory calculation of electron beams is the basis of parametric evaluation of vacuum electron devices. The interaction between electron trajectory and electric field makes the calculation of trajectory a complicated electro-magnetic particle coupling problem. This paper proposes a physics-driven calculation method for electro-magnetic particle coupling process in vacuum electron devices. The physics-driven calculation method for electromagnetic field incorporating space charge effect can be utilized repeatedly after training, and the node-based computation approach eliminates the node mapping between coupling fields, providing a novel approach to avoid redundant mesh regeneration and recalculations.

Index Terms—Coupling, Physics-Informed Neural Networks, Trajectory tracking, Electron devices.

I. INTRODUCTION

THE OPERATION of Vacuum Electronic Devices (VEDs) relies on the quality of electron beams [1]. The calculation of electron trajectories is a coupling process with electromagnetic fields involved. Current mesh-based methods are resource-intensive when dealing with complicated geometric structures and changing space charge distribution [2]. To address these challenges, a physics-driven coupling framework is proposed. PINN-based computation models of static magnetic and electric field with space charge incorporated are trained and utilized repeatedly to update electric field and trajectory distribution, leading to steady state trajectory results.

II. FORMATION OF ELECTRON BEAMS

The electron beam is central to VED performance and is formed as electrons emitted from the cathode, accelerated and compressed by electromagnetic field to form high-speed helical beams. Electrons can be regarded as moving in static fields. The electrostatic field can be formulated by Poisson's equation:

$$\nabla^2 u = -\frac{\rho}{\varepsilon} \quad (1)$$

where u is the electric potential, ρ is the charge density, and ε is the permittivity. Similar with electric field, magnetic field calculation takes magnetic vector potential (MVP) as an intermediate vector field of interest. The governing equation after simplifying with Coulomb gauge can be expressed as:

$$\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J} \quad (2)$$

where μ_0 is the permeability of free space, and $\mathbf{J}$ is the current density inside the coil.

Under the joint action of applied electromagnetic field, the force on the electron follows the Lorentz force equation:

$$\mathbf{F} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \quad (3)$$

which can be decomposed to differential equations of electron motion for trajectory calculation.

The high electron density in beams makes the space charge effect significant. Electron motion causes dynamic change of

charge distribution, which modifies the potential and affects trajectory subsequently, resulting in a highly nonlinear and dynamically coupled Multiphysics problem.

III. PHYSICS-DRIVEN COUPLING CALCULATION METHOD

A. Static Electromagnetic Field Calculation Considering Space Charge Effect

To address complex VED geometries without mesh regeneration, PINN is adopted for electromagnetic field computation. As shown in Fig. 1, the electric and magnetic networks take spatial coordinates (z, r) and corresponding charge density ρ as input to predict the field distributions.

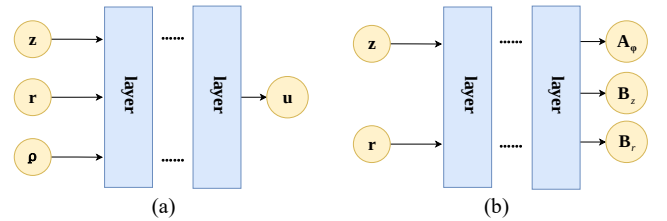


Fig. 1. PINN structure of (a): electric field and (b) magnetic field.

Due to the similar forms of (1) and (2), both fields are solved with Vanilla PINN[4], with tailored sampling strategies and loss function to account for domain and physics differences.

For electric field, piecewise modeling and sampling methods are taken to address the angular points and curved surface of electrodes. Internal points are mesh-based and enhanced with adaptive sampling during training to ensure adequate coverage in areas of complex geometry or poor performance.

While cylindrical coordinates and axisymmetry simplify the problem, they introduce singularities near the axis, requiring equation transformation to eliminate fractional terms. Scaling is also applied to address data disparity, particularly in the electric field. The loss function is given by:

$$\mathcal{L}_E = MSE_{E-pde} + MSE_{E-bc} + MSE_{E-ic} \quad (4)$$

$$MSE_{E-pde} = \frac{1}{N_{ini} + N_{adp}} \sum_{i=1}^{N_{ini} + N_{adp}} \left| \tilde{f}_E(u(z, r, \rho)) \right| \quad (5)$$

where bc means boundary condition, and ic means initial condition. N_{ini} and N_{adp} are the number of collocation points

from initial sampling and adaptive sampling, $\tilde{f}_E$ is the governing pde equation after scaling.

The magnetic field loss function requires no scaling, but different sampling is used for source and source-free regions. Also, to reduce errors from parameter gradients, the field is output jointly with MVP.

B. Electron Trajectory and Charge Density Calculation

Electron motion in the electromagnetic field is derived from (3). Since the period of electron motion is constantly being compressed, 4th-order Runge-Kutta method with adaptive step size is employed for more accurate trajectory calculation. The change of step size is adjusted by velocity r-component.

Considering the space charge effect, charge density induced by electrons with structure factor considered is formulated as:

$$\rho(n, r, z, t) = - \frac{\left(r_1 + r_1 + \frac{n}{5} \Delta r_{t_0} \right) I_b}{\left(r_1 + r_1 + \Delta r_{t_0} \right) 2\pi r_n \Delta r_{t_n} v_z} \quad (6)$$

where n is the serial number of electrons in beam, with the lowest electron labeled as particle 1. I_b is the beam current, Δr is the average distance in r direction between emission points at time t , and v_z is velocity component in z -direction.

C. Coupling Calculation Considering Space Charge

The coupling of electro-magnetic particle fields can be achieved by passing changing distribution of space charge, the process of coupling calculation can be summarized as Fig. 2.

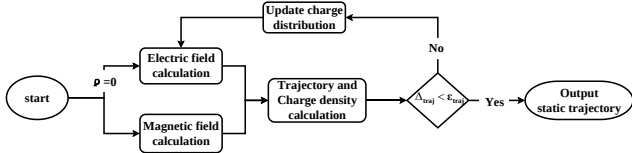


Fig. 2. Workflow of coupling calculation.

The electric field without space charge and magnetic field are first calculated to determine the electron trajectory under corresponding fields. The space charge distribution is then updated based on the trajectory, followed by recalculating the electric field. This iterative process continues until steady-state electron trajectories are archived.

In mesh-based methods, node mapping is required for coupling whereas PINN-based meshless methods can replace collocation points in relevant regions directly. When training electric field, transition layers are added for charge density continuity as shown in Fig. 3.

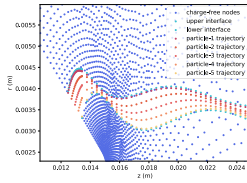


Fig. 3. E field and trajectory sample points replacement.

IV. NUMERICAL SIMULATIONS

The magnetron injection gun was used for validation. Due to experiment platform limitations, a CST simulation model was built for comparison, the 2D cross-section is shown as Fig. 4.

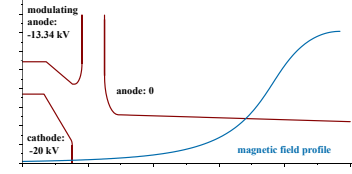


Fig. 4. 2D magnetron injection model for proposed calculation method.

The electric field PINN used 8056 sampled nodes and a 5-layer architecture with 64 neurons per layer, while the magnetic field PINN had 3 hidden layers and 4004 nodes. Field distributions in the source-free region are shown in Fig. 5, and the MSEs compared with FEM are 1.53e-2 and 1.05e-3 respectively.

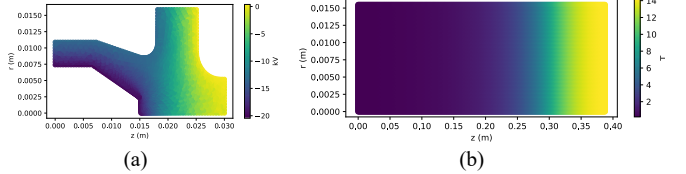


Fig. 5. Field distribution calculated with PINN model (a): electric potential, (b) component magnetic field of z direction.

Calculation of trajectory and charge density distribution after iterations are shown in Fig. 6. Compared to CST, the proposed method reduces computation time by 85.93%, mainly in electromagnetic field computation.

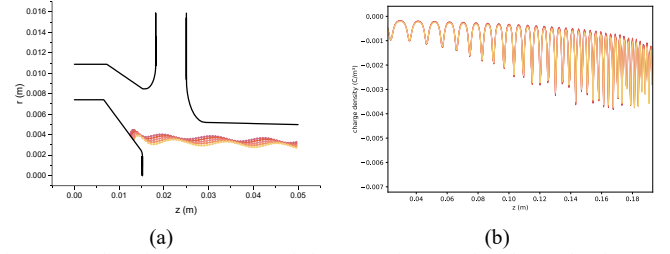


Fig. 6. (a) r -direction component of electron trajectory, (b): charge density distribution along z axis.

V. CONCLUSION

A physics-driven coupling calculation method is proposed for VEDs to calculation electron trajectory. PINN is utilized to solve the distribution of electro-magnetic fields. The electric field model with space charge effect considered can be employed without repeated training, significantly reducing computational cost. Also, the meshless nature of PINN simplifies the node mapping between coupled fields, facilitating more efficient and streamlined iterative computations.

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