

Harmonic-domain Model Order Reduction of MQS Problem by Spectral POD

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For transformers under periodic non-sinusoidal excitation, the harmonic balance method is a widely adopted approach for solving the magnetic field. However, performing model order reduction (MOR) in harmonic-domain analysis is a problem, as traditional MOR methods which work in the time domain are ineffective in harmonic-domain analysis. To address this issue, this paper introduces a spectral proper orthogonal decomposition (spectral POD) within the harmonic balance scheme for magneto-quasi-static (MQS) field problems. This approach effectively reduces the degrees of freedoms (DoFs) for computation of each harmonic, and significantly improves computational efficiency. Additionally, the methodologies for selecting snapshots and processing samples are discussed in this paper. To demonstrate the implementation of this MOR approach, a transformer model with excitations containing 49 harmonic orders is presented as a case study. The correctness and accuracy of the proposed MOR approach are validated by comparing its results with those obtained from full-order harmonic balance computations.

Index Terms—Harmonic domain, model order reduction, periodic non-sinusoidal excitation, spectral POD.

I. INTRODUCTION

HARMONIC BALANCE METHOD is a widely used approach for analyzing the magnetic field of transformers under periodic non-sinusoidal excitation [1]. However, the computational cost remains a significant challenge due to the large scale and intricate structure of transformers. Incorporating a MOR approach within the harmonic balance scheme provides a cost-effective solution to improve computational efficiency in magnetic field analysis.

Traditional MOR approaches operating in the time domain, such as POD [2], machine learning-based models, and dynamic mode decomposition, often do not work when applied to harmonic-domain studies. To address this issue, this paper is based on the key characteristic of the harmonic balance method—namely, the independent computation of each harmonic—and introduces spectral POD within the harmonic balance framework. This method enables the solution of unknown variables for each harmonic within a reduced subspace, thereby significantly reducing computational complexity.

II. MAGNETO-QUASI-STATIC FIELD

In transformers, the winding current is a non-sinusoidal waveform, including harmonic currents from the 1st to the 49th order [3]. Under this condition, the magnetic field of the transformer is treated as a MQS field problem, as expressed:

$$\nabla \times \frac{1}{\mu} (\nabla \times \mathbf{A}(t)) + \sigma \frac{\partial \mathbf{A}(t)}{\partial t} = \mathbf{J}(t), \quad (1)$$

where μ is the permeability; σ is the conductivity, t is the time, $\mathbf{A}$ is the magnetic vector potential, and $\mathbf{J}$ is the current density.

Using the harmonic balance method, the spatially-discretized equation in harmonic domain is as follows:

$$\mathbf{M}\mathbf{A}_k + j\omega k \mathbf{K}\mathbf{A}_k = \mathbf{C}\mathbf{J}, \quad (2)$$

where $\mathbf{M}$ and $\mathbf{K}$ are a reluctivity coefficient matrix and a

conductivity matrix in the dimension N_s (N_s is the DoF of this system), and $\mathbf{C}$ is a matrix depending on the geometrical features of the windings and turns' number. Besides, $k \in \mathbb{N}^+$ represents harmonic at k -th order.

III. MOR IN HARMONIC DOMAIN

It is clear that, using the harmonic balance method, the steady-state responses of different harmonics are solved independently. Based on this computational characteristic, this MOR approach incorporating spectral POD within the harmonic balance scheme is put forward, enabling each harmonic to be computed within its corresponding reduced subspace.

A. Snapshots Selection

As an *a posteriori* method, this MOR approach relies on precomputed results as samples to approximate unknown variables within a reduced subspace. These samples are selected from the time domain [4] and must exhibit behaviors similar to those subsequent computations. Specifically, similar behavior is defined as conditions where the samples are obtained within the same range of harmonic orders as the subsequent computations or under excitations that include the harmonics involved in the subsequent computations.

As shown in Fig.1, the precomputed results provide time-domain snapshots in the form of $[\mathbf{A}(t_1), \mathbf{A}(t_2), \mathbf{A}(t_3), \dots]$, where $\mathbf{A}(t_k)$ represents the magnetic vector potential at time t_k . These snapshots are then organized into distinct groups $[\mathbf{G}_1, \mathbf{G}_2, \mathbf{G}_3, \dots, \mathbf{G}_n]$ by time marching, so that each group $\mathbf{G}_k$ contains time-domain results of multiple time points. It is important to note that the number of time points included in each group $\mathbf{G}_k$ must exceed the number of involved harmonic orders.

By applying the fast Fourier transform (FFT), each group is transformed into the frequency domain, resulting in a complex number matrix $[\mathbf{G}_1, \mathbf{G}_2, \mathbf{G}_3, \dots, \mathbf{G}_n]$. In this representation, each

$\mathcal{G}_{h\ell}$ contains data spanning all the harmonic orders involved. As demonstrated in (2), the harmonic balance method allows for the independent computation of different harmonic orders. To reduce the computational cost associated with each harmonic order, it is necessary to generate a low-dimensional subspace for each harmonic order. To achieve this, a sample matrix $\mathcal{G}_{h\ell}$ is constructed by selecting the ℓ -th harmonic order from $[\mathcal{G}_1, \mathcal{G}_2, \mathcal{G}_3, \dots, \mathcal{G}_n]$. Here, $\mathcal{G}_{h\ell}$ would serve as the sample matrix to generate the subspace for ℓ -th harmonic order.

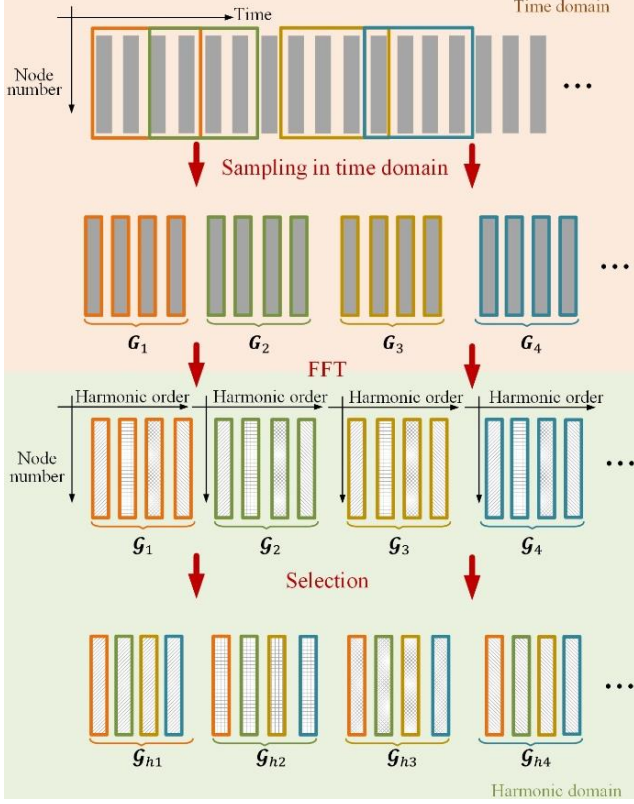


Fig.1. Snapshot processing for model reduction in harmonic balance scheme.

B. MOR

Unlike traditional POD computations that employ a fixed subspace for the entire time-step computation, the spectral POD in the harmonic balance scheme utilizes multiple subspaces to solve unknown variable. Taking the computation of ℓ -th order harmonic as example, the orthogonal basis of subspace is obtained by performing singular value decomposition on the sampling matrix $\mathcal{G}_{h\ell}$

$$\mathcal{G}_{h\ell} = \mathbf{P} \begin{bmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \ddots \end{bmatrix} \mathbf{V}^H \quad (3)$$

where $\mathbf{P}$ is an $N_s \times N_d$ matrix containing N_d orthogonal basis vectors, and $\mathbf{V}$ is an orthonormal matrix. The diagonal matrix $\mathbf{\Sigma} = \text{Diag}(\lambda_1, \lambda_2, \dots, \lambda_{N_d})$ contains N_d singular values that capture the dominant energy of the system, namely $N_d \ll N_s$.

Based on an orthogonal basis, the unknown variable of magnetic vector potential in the ℓ -th order harmonic, $\mathcal{A}_{h\ell}$, is approximated as [5]

$$\mathcal{A}_{h\ell} = \mathbf{P} \alpha_{h\ell}, \quad (4)$$

where $\alpha_{h\ell} = [\alpha_1, \dots, \alpha_{N_d}]^T$ is the coordinate in the reduced subspace.

Substituting (4) in (2) and left-multiplying $\mathbf{P}^H$ (the conjugate transpose of $\mathbf{P}$) to both sides yield the reduced-order governing equation:

$$\mathbf{P}^H (\mathbf{M} + j\omega_{h\ell} \mathbf{K}) \mathbf{P} \alpha_{h\ell} = \mathbf{P}^H \mathbf{C} \mathbf{J} \quad (5)$$

For each order computation, DoFs are reduced from N_s to N_d , significantly lowering the computational burden

IV. RESULTS AND DISCUSSION

A transformer model is developed to investigate the flux density under non-sinusoidal current excitation. The sample set is generated using a random function incorporating 49 harmonic orders and comprises 200 snapshots. Utilizing the same set of samples, the flux density of two scenarios is analyzed and compared against the results obtained from the full-order model, as Fig.2. Besides, in one scenario, the full-order model requires 7.9 seconds to compute one cycle, whereas the MOR approach based on spectral POD within the harmonic balance scheme completes the same task in 0.2 seconds, reducing the computational time to 2.5% as the degrees of freedom are reduced from 9762 to 4.

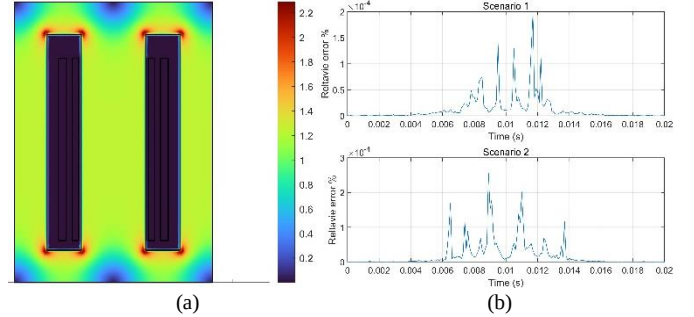


Fig.2. (a) Flux density distribution by MOR (T). (b) Error comparisons.

V. CONCLUSION

In this paper, a MOR approach based on spectral POD within the harmonic balance scheme is proposed to investigate the magnetic field of transformers under periodic non-sinusoidal current excitation. To evaluate the correctness and efficiency of this method, a transformer model is constructed, and its results are compared with those of the full-order model.

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