

An Effective Surface Impedance in a Magnetic Scalar Potential Formulation

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The aim is to avoid expensive computations of eddy currents in nonlinear 3D problems which are large compared to the penetration depth. To this end a nonlinear effective surface impedance is introduced. The effective surface impedance is determined by solving a suitable cell problem for the case of a time-harmonic eddy current problem. With this effective surface impedance an equivalent static magnetic field problem can be solved efficiently using a magnetic scalar potential in finite element models. The method with the effective material is cheap and accurate compared to the eddy current problem as shown by numerical simulations.

Index Terms—Cell problem, eddy current problem, effective surface impedance boundary condition, finite element method, magnetic scalar potential.

I. INTRODUCTION

Surface impedance boundary conditions (SIBCs) are often utilized to save computational costs in simulations of eddy current problems (ECPs) in massive iron parts, where the electromagnetic fields perpendicular to the surface of the iron vary strongly compared to the tangential direction. For a proper application of SIBCs several aspects need to be considered like the curvature of the surface, singularities along edges and in corners, etc., see for example [1] and [2], respectively. In this work, the focus is on a SIBC for nonlinear materials and its integration into a scalar potential formulation for the finite element method (FEM). We propose to compute an effective surface impedance (ESI) solving a suitable cell problem (CP). This ESI accurately reflects the nonlinear behavior of the material given by a BH-curve. The ESI is integrated in an essentially static magnetic field problem (SMFP) for the FEM. To demonstrate the capability of the proposed method, a typical numerical problem is studied. The method provides very accurate results.

II. THE METHODS

Although the material properties are nonlinear, for the sake of simplicity we assume the electromagnetic fields to be time-harmonic, i.e. of the form $\mathbf{u}(\mathbf{x}, t) = \text{Re}\{\hat{\mathbf{u}}(\mathbf{x})e^{j\omega t}\}$, where the hat stands for complex-valued functions, $\text{Re}\{\cdot\}$ denotes the real part of complex numbers, $\mathbf{x}$ a point in space, t the time, j the imaginary unit and ω the angular frequency. The hat symbol is omitted below to simplify the representation. The derivative with respect to time becomes a simple multiplication, i.e., $\partial_t \mathbf{u} = j\omega \mathbf{u}(\mathbf{x}, t)$.

A. The $T, \Phi - \Phi$ Formulation

The $T, \Phi - \Phi$ formulation with the current vector potential (CVP) T and the reduced magnetic scalar potential (RMSP) Φ is used to solve the basic ECP, which also serves as a reference solution (RS) in III. A detailed explanation of this formulation can be found, for example, in [3]. The following is a summary

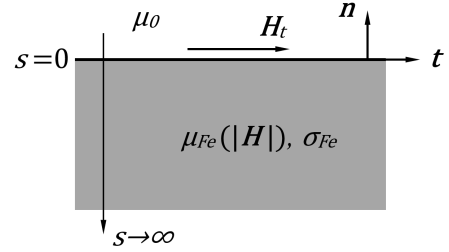


Fig. 1. Cell problem: nonlinear conducting infinite half space.

of the nonlinear ECP in the frequency domain, as required for this work.

The weak form of the ECP for the FEM reads:

$$\begin{aligned} \text{Find } (T_h, \Phi_h) \in V_{Dh} := \{(T_h, \Phi_h) : T_h \in \mathcal{U}_h, \Phi_h \in \mathcal{V}_h \text{ and } T_h \times \mathbf{n} = \mathbf{0} \text{ on } \Gamma_{0c} \cup \Gamma_{Hc}, \Phi_h = \Phi_D \text{ on } \Gamma_{Dh}\}, \text{ such that} \\ \int_{\Omega_c} \rho \text{curl } T_h \cdot \text{curl } v_h d\Omega + j\omega \int_{\Omega_c} \mu (T_h - \text{grad } \Phi_h) \cdot v_h d\Omega = \\ - \int_{\Gamma_{c0}} (\rho \text{curl } T_h \times \mathbf{n}) \cdot v_h d\Gamma = 0 \quad (1) \\ j\omega \int_{\Omega} \mu \text{grad } \Phi_h \cdot \text{grad } v_h d\Omega - j\omega \int_{\Omega_c} \mu T_h \cdot \text{grad } v_h d\Omega = 0 \end{aligned}$$

for all $(v_h, v_h) \in V_{0h}$, where $\mathcal{U}_h$ and $\mathcal{V}_h$ are finite element subspaces of $H(\text{curl}, \Omega_c)$ and $H^1(\Omega)$, $\Gamma_{c0} = \Omega_c \cap \Omega_0$ is the interface between the conducting domain Ω_c and the non-conducting one Ω_0 , ρ is the resistivity, μ the magnetic permeability of iron and μ_0 that of air, respectively. The surface term in (1) is equal to zero and is present to justify the explanation of (6).

B. Cell Problem

The CP is the infinite conducting half space with nonlinear iron $\mu(|H|)$, see Fig. 1. We compute the single component magnetic field strength H with $H_t = Ht$ solving

$$\rho \partial_{ss} H + j\omega \mu(|H|) H = 0 \quad (2)$$

with $H = H_0$ prescribed at $s = 0$ in one dimension on a sufficiently large interval $I = [s = 0, s \rightarrow \infty]$. The nonlinear

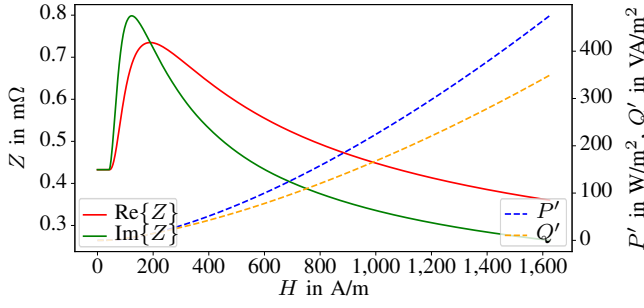


Fig. 2. Effective surface impedance $Z(H_0)$ with the losses P' and the fictitious reactive power Q' per unit area of the cell problem.

material $\mu(|H|)$ is given by a known BH-curve [4]. The problem (2) is solved for different boundary conditions H_0 . The losses and the fictitious reactive power per unit area denoted by the prime symbol,

$$P' = \frac{1}{2} \int_I E J^* ds \quad \text{and} \quad Q' = \frac{\omega}{2} \int_I H B^* ds, \quad (3)$$

respectively, are calculated in the half space, compare with Fig. 2, where E is the electric field strength, J the current density and the asterisk symbol denotes conjugate complex. Considering the Poynting's theorem, the relations

$$(P' + jQ')A = S_0 A = E_0 H_0^* A = Z H_0 H_0^* A = Z H_0^2 A \quad (4)$$

hold, where A is a unit area and S_0 the Poynting vector. The index 0 denotes quantities evaluated at $s = 0$. The ESI can thus be calculated by

$$Z = \frac{S_0}{H_0^2} \quad (5)$$

and is shown in Fig. 2.

C. Static Magnetic Field with Effective Surface Impedance

Now, imagine the conducting domain Ω_c in (1) is removed and the essentially static magnetic field remains in air Ω_0 . In order to obtain the same static magnetic field, the influence of the material in Ω_c is considered by a suitable boundary condition on Γ_{c0} . Rewriting the surface term in (1), which originally results from integration by parts in the $T, \Phi - \Phi$ formulation, so that Φ occurs only, yields

$$\int_{\Gamma_{c0}} \rho \operatorname{curl} T \times n \cdot v d\Gamma = \int_{\Gamma_{c0}} Z \operatorname{grad}_t \Phi \cdot \operatorname{grad}_t v d\Gamma \quad (6)$$

with the tangential derivative $\operatorname{grad}_t(\cdot) = t \cdot \operatorname{grad}(\cdot)$. This leads to the weak form for the SMFP with the ESI Z :

Find $\Phi_h \in V_{Dh} := \{\Phi_h : \Phi_h \in \mathcal{V}_{Dh} \text{ and } \Phi_h = \Phi_0 \text{ on } \Gamma_{H_D}\}$, such that

$$j\omega \int_{\Omega_0} \mu_0 \operatorname{grad} \Phi_h \cdot \operatorname{grad} v_h d\Omega - \int_{\Gamma_{c0}} Z \operatorname{grad}_t \Phi \cdot \operatorname{grad}_t v_h d\Gamma d\Omega = 0 \quad (7)$$

for all $v_h \in V_{0h}$, where $\mathcal{V}_h$ is a finite element subspace of $H^1(\Omega_0)$.

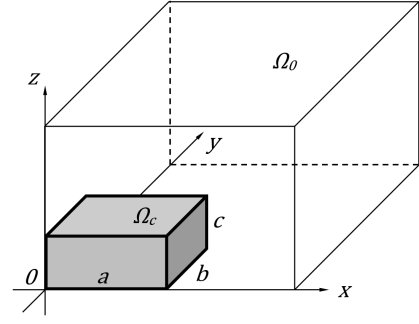


Fig. 3. One eighth of the numerical example, the gray part represents iron, $a = b = 0.5$ m and $c = 0.2$ m, and $x = 0$, $y = 0$, and $z = 0$ are planes of symmetry, 1 m is added to the iron in each direction, resulting in the air.

III. NUMERICAL EXAMPLE

The numerical example shown in Fig. 3 is studied to evaluate the performance of the proposed method. The proposed SMFP method with the ESI is compared with the ECP as a RS. The parameters chosen for the RS are the conductivity of iron $\sigma = 2.08 \cdot 10^6$ S/m, the permeability of iron $\mu(|H|)$ based on the measured BH-curve [4] and $\mu_0 = 4\pi \cdot 10^{-7}$ Vs/(Am) of air and the frequency $f = 50$ Hz. The ESI Z in Fig. 2 is used in the SMFP. The boundary conditions $\Phi_D(z = 0) = 0$ and $\Phi_D(z_{max}) = \Phi_0$ are prescribed for the ECP as well as for the SMFP. From the RS using (1)

$$P_{RS} + jQ_{RS} = \frac{1}{2} \int_{\Omega_c} E \cdot J^* d\Omega + \frac{j\omega}{2} \int_{\Omega_c} H \cdot \mu(|H|) H^* d\Omega \quad (8)$$

is determined. To obtain $P_{ESI} + jQ_{ESI}$, the curves P' and Q' in Fig. 2 and $H = |\operatorname{grad}_t \Phi|$ are used based on the solution of (7) on Γ_{0c} . Some results are summarized in Tab. I. The agreement of the losses and reactive powers is very satisfactory.

TABLE I
RESULT DATA ¹⁾

Φ_0 A	P_{RS} W	P_{ESI} W	ϵ_P %	Q_{RS} VA	Q_{ESI} VA	ϵ_Q %
1,000	1.34	1.344	0.51	1.534	1.547	0.86
2,000	6.614	6.62	0.08	5.997	6.019	0.36
4,000	40.98	41.02	0.08	31.35	31.42	0.25

¹⁾ Result Data are valid for one eighth of the numerical problem, ϵ_P relative error in P , ϵ_Q relative error in Q .

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