

Towards Purely Parameter-Specific Model Order Reduction: A Tensor Decomposition-Based Approach

Ze Guo^{1,2,3} and Zuqi Tang¹

¹Univ. Lille, Arts et Métier Institute of Technology, Centrale Lille, Junia, ULR 2697-L2EP, F-59000 Lille, France, ze.guo.etu@univ-lille.fr; zuqi.tang@univ-lille.fr

²Institute of Electrical Engineering, Chinese Academy of Sciences, Beijing 100190, China

³University of Chinese Academy of Sciences, Beijing 100190, China

In the context of digital twins, projection-based model order reduction (MOR) plays a crucial role in real-time control, fault diagnosis and performance prediction, particularly when accounting for multi-parameter variations in complex systems. However, traditional MOR techniques often exhibit limited robustness against parameter variations. In this work, a parameter-specific MOR approach is proposed for nonlinear multi-parameter electromagnetic problems based on tensor decomposition. The approach is compared with classical MOR as well as local MOR, typically reliant on clustering techniques, through different numerical examples such as an electrical motor with four-parameter. The numerical examples show that the proposed approach outperforms classical MOR and local MOR methods in terms of computational efficiency and accuracy.

Index Terms—Electromagnetic problem, model order reduction, multi-parameter, tensor decomposition.

I. INTRODUCTION

ELECTRIC EQUIPMENTS are the core component of power production, transmission, and conversion. Ensuring their safe and stable operation is crucial, which has driven increasing focus on real-time control, fault diagnosis, and performance prediction within the digital twin framework.

Developing a digital twin model capable of predicting the behavior of electrical equipment under various operating conditions requires a substantial amount of high-fidelity data that spans a wide range of scenarios. Since experimental data are often expensive and not readily available, a more feasible approach is to generate such data through multi-parameter numerical simulations. However, the inherent non-linearity and variability of different operating conditions make repeated simulations computationally expensive.

To alleviate the computational burden of repeated simulations, projection-based model order reduction (MOR) techniques, such as Proper Orthogonal Decomposition (POD) [1], are widely used to reduce computational costs by replacing high-fidelity models with low-dimensional reduced-order models. However, traditional MOR methods are ineffective for nonlinear multi-parameter problems, as the global reduced-order basis generated by these methods often lacks robustness with respect to parameter variations [2].

To address this challenge, several advanced local MOR methods have been introduced, such as clustering POD [2] and block POD [3]. These methods partition the snapshots and assign separate reduced bases to each local sub-region, thereby enhancing the robustness of the reduced basis in capturing local characteristics. Nevertheless, these local MOR methods face challenges when system characteristics vary significantly with the parameters. This issue arises from the assumption of local similarity, which assumes that the essential physics of the governing equations can be preserved independently in different regions. However, strong interactions between

regions can violate this assumption and reduce model accuracy. Furthermore, parameter partitioning, typically based on empirical or greedy methods, is often case-specific and difficult to generalize. The process is further complicated by the lack of prior knowledge during partitioning and inconsistent scaling of parameters.

Alternatively, a hybrid approach combining tensor decomposition with MOR (TDMOR) is proposed [4] to construct parameter-specific MOR for multi-parameter electromagnetic problems. In this work, a comprehensive comparison is conducted among the proposed TDMOR, classical MOR, and conventional local MOR methods. Numerical examples demonstrate that our proposed approach provides a more localized reduced-order basis, enhancing model accuracy and ensuring the parameter-specific characteristics of the MOR for multi-parametric problems.

II. METHODOLOGY

This section begins with a brief introduction to the definition of multi-parameter electromagnetic problems. Then, a concise review of the projection-based MOR technique (typically with classical POD) is provided. Finally, the implementation of local MOR and proposed TDMOR is illustrated, respectively.

A. Multi-parameter electromagnetic problems

Considering the diverse operating conditions of the electric equipment, the nonlinear electromagnetic problem of the electric equipment with multiple parameters can be simplified to the following system by finite element (FE) discretization:

$$\mathbb{L}(\mathbf{u}, \mathbf{p})\mathbf{u}(\mathbf{p}) = \mathbf{F}(\mathbf{p}) \text{ with } \mathbf{p} = (p_1, p_2, \dots, p_D). \quad (1)$$

Here, $\mathbb{L}$ denotes the FE system matrix associated with the D -dimensional parameter vector $\mathbf{p}$, consisted by different parameters. $\mathbf{u}$ corresponds to the discretized unknowns of the FE system, such as the magnetic vector potential, while $\mathbf{F}$ is

the right-hand side vector derived from the discretization of the source term.

Considering the effects of magnetic saturation, the nonlinear process of solving (1) for various parameter $\mathbf{p}$ is time-consuming. Therefore, projection-based MOR, typically with POD, is employed to alleviate the computational burden.

B. Classical MOR

The classical projection-based MOR consists in building a global reduced basis by sampling the parameter space $[\mathbf{p}_1, \mathbf{p}_2, \dots, \mathbf{p}_{n_s}]$ and generating a snapshot matrix $\mathbf{S}_{\text{POD}} = [\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_{n_s}]$ of dimension $m \times n_s$ by solving system (1), with n_s the number of total snapshots defined as $n_s = n_1 \times n_2 \times \dots \times n_D$, where n_i is the sampling number of the i -th parameter, and m the Degree of Freedom (DoFs) of (1). By singular value decomposition (SVD), the snapshot matrix $\mathbf{S}_{\text{POD}}$ can be written as:

$$\mathbf{S}_{\text{POD}} = \mathbf{U}_{\text{POD}} \mathbf{\Sigma}_{\text{POD}} \mathbf{V}_{\text{POD}}^T \text{ with } \mathbf{U}_{\text{POD}} \mathbf{U}_{\text{POD}}^T = \mathbf{I}. \quad (2)$$

The left singular vectors (columns of $\mathbf{U}_{\text{POD}}$) can be used as the global reduced basis. Once these steps have been completed during the offline stage, in the online stage, for an arbitrary parameter $\mathbf{p}$, the reduce problem reads:

$$\mathbf{U}_{\text{POD}}^T \mathbb{L}(\mathbf{u}, \mathbf{p}) \mathbf{U}_{\text{POD}} \mathbf{U}_{\text{POD}}^T \mathbf{u}(\mathbf{p}) = \mathbf{U}_{\text{POD}}^T \mathbf{F}(\mathbf{p}). \quad (3)$$

C. Local MOR (LMOR)

Similar to the classical MOR, during the offline stage, local MOR clusters the snapshots based on specific parameters or system states, constructing several different snapshot matrices denoted as $\mathbf{S}_{\text{LMOR}} = [\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_{n_p}]$, with dimensions $m \times n_p$, where n_p represents the number of snapshots within each local sub-region. The reduced basis $\mathbf{U}_{\text{LMOR}}$ for a given sub-region is then obtained through SVD, as outlined in (2). Then, the reduced problem is solved by switching between different local reduced bases as needed:

$$\mathbf{U}_{\text{LMOR}}^T \mathbb{L}(\mathbf{u}, \mathbf{p}) \mathbf{U}_{\text{LMOR}} \mathbf{U}_{\text{LMOR}}^T \mathbf{u}(\mathbf{p}) = \mathbf{U}_{\text{LMOR}}^T \mathbf{F}(\mathbf{p}). \quad (4)$$

D. TDMOR

For TDMOR, all the n_s snapshots are organized in a $D+1$ dimensional tensor, of size $m \times n_1 \times n_2 \times \dots \times n_D$, such as:

$$(\Phi)_{:,j_1,\dots,j_D} = \mathbf{u}(p_1^{j_1}, \dots, p_D^{j_D}). \quad (5)$$

With the Tensor decomposition, a low rank approximation $\tilde{\Phi}$ of the full tensor Φ can be obtained:

$$\tilde{\Phi} = \sum_{j=1}^{\tilde{m}} \sum_{q_1=1}^{\tilde{n}_1} \dots \sum_{q_D=1}^{\tilde{n}_D} (\mathbf{C})_{j,q_1,\dots,q_D} \mathbf{w}^j \circ \sigma_1^{q_1} \circ \dots \circ \sigma_{D-1}^{q_{D-1}} \circ \sigma_D^{q_D}, \quad (6)$$

where $\tilde{m}, \tilde{n}_1, \dots, \tilde{n}_D$ referred to the compressed dimension much smaller than $m, n_1, \dots, n_D$. $\mathbf{C}$ is a core tensor of dimension less than the full tensor Φ . TDMOR constructs parameter-specific reduced basis $\mathbf{U}_{\text{TD}}(\mathbf{p})$ in two stages. Specifically, at the offline stage, a global orthogonal basis $\mathbf{U}_g$ and a series of matrices $\mathbf{S}_i$ with $1 \leq i \leq D-1$, which encodes information on the relationship between various parameters, can be obtained according to (6)

$$\mathbf{U}_g = [\mathbf{w}^1, \dots, \mathbf{w}^{\tilde{m}}] \in \mathbb{R}^{m \times \tilde{m}}, \quad (7)$$

$$\mathbf{S}_i = [\sigma_i^1, \dots, \sigma_i^{\tilde{n}_i}]^T \in \mathbb{R}^{\tilde{n}_i \times n_i}. \quad (8)$$

At the online stage, a parameter specific core matrix $\mathbf{C}_e(\mathbf{p})$ can be obtained with the core tensor $\mathbf{C}$, matrices $\mathbf{S}_i$ and the following extraction-interpolation procedure:

$$\mathbf{C}_e(\mathbf{p}) = \mathbf{C} \times_2 (\mathbf{S}_1 \mathbf{e}^1(\mathbf{p})) \dots \times_D (\mathbf{S}_{D-1} \mathbf{e}^{D-1}(\mathbf{p})), \quad (9)$$

where $\mathbf{e}^k$ ($1 \leq k \leq D-1$) is the coordinate vector at specific parameter $\mathbf{p}$ obtained by Lagrangian interpolation. The SVD is only needed to apply to the small size core matrix $\mathbf{C}_e(\mathbf{p})$, ensuring that the subsequent online basis update process is "on the fly":

$$\mathbf{C}_e(\mathbf{p}) = \mathbf{U}_c(\mathbf{p}) \mathbf{\Sigma}_c(\mathbf{p}) \mathbf{V}_c^T(\mathbf{p}). \quad (10)$$

Finally, a parameter-specific reduced basis $\mathbf{U}_{\text{TD}}(\mathbf{p})$ can be obtained as:

$$\mathbf{U}_{\text{TD}}(\mathbf{p}) = \mathbf{U}_g \mathbf{U}_c(\mathbf{p}). \quad (11)$$

III. NUMERICAL EXAMPLE

A preliminary result, illustrating the application of the proposed TDMOR approach to an electric motor with four parameters, is shown in Fig. 1. This result demonstrates that TDMOR achieves significantly better performance compared to classical MOR and local MOR. A more detailed comparison of the different approaches will be presented in the full paper.

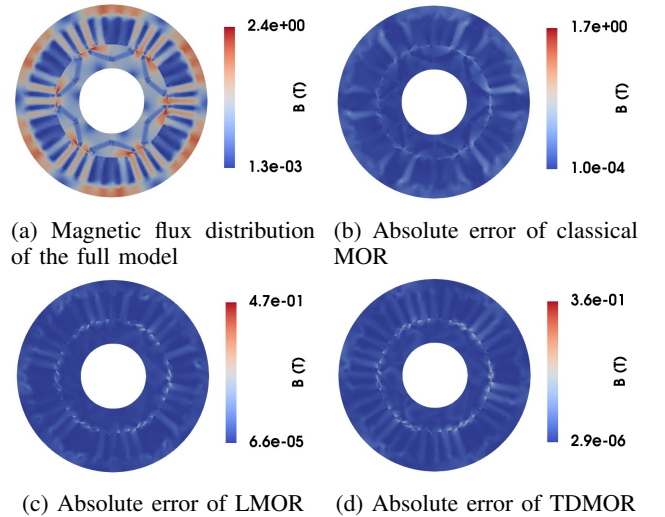


Fig. 1: Accuracy comparison of different MOR approaches.

REFERENCES

- [1] D. Schmidthäusler and M. Clemens, "Low-order electroquasistatic field simulations based on proper orthogonal decomposition," *IEEE Transactions on Magnetics*, vol. 48, no. 2, pp. 567–570, 2012.
- [2] D. Amsallem, M. J. Zahr, and C. Farhat, "Nonlinear model order reduction based on local reduced-order bases," *International Journal for Numerical Methods in Engineering*, vol. 92, no. 10, pp. 891–916, 2012.
- [3] T. Shimotani, Y. Sato, T. Sato, and H. Igarashi, "Fast finite-element analysis of motors using block model order reduction," *IEEE Transactions on Magnetics*, vol. 52, no. 3, pp. 1–4, 2016.
- [4] Z. Guo, Z. Tang, and Z. Ren, "Tensor decomposition-based model order reduction applied to multi-parameter electromagnetic problems in the context of digital twins," *High Voltage*, 2025.