

Magnetic Field Conforming Formulations for Foil Windings

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In this contribution, we extend the foil winding homogenization method to magnetic field conforming formulations. First, we propose a full magnetic field foil winding formulation by analogy with magnetic flux density conforming formulations. Second, we introduce the magnetic scalar potential in non-conducting regions to improve the efficiency of the model. The proposed models are validated on a benchmark problem.

Index Terms—Approximation methods, finite element analysis, foil winding, inductors

I. INTRODUCTION

FOIL WINDINGS require specific modeling techniques, since the explicit discretization of all turns in a finite element model is computationally expensive, particularly for 3D applications. To overcome this issue, homogenization techniques [1], [2] have been proposed, based on magnetic flux density conforming formulations. Recently, some contributions have addressed the extension to the time-domain [3], the stability [4], and the application to superconducting systems [5] of these formulations. In this contribution, we propose a magnetic field conforming h -formulation for the homogenization of foil windings, derived by analogy with the a - v -formulation already documented in the literature. The magnetic scalar potential is also introduced in non-conducting regions to reduce the number of degrees of freedom required to solve the problem, which leads to the h - ϕ formulation for foil windings. The proposed models are validated on a superconducting benchmark presented in [5], and their efficiency is compared to state-of-the-art foil winding formulations.

II. FINITE ELEMENT FORMULATIONS

We consider a coil of N_c thin conductors (or *foils*), constituting the conducting subset Ω_c of the computational domain Ω , whose cross-section (α - β plane) is shown in Fig. 1. The current in each conductor Ω_{c_i} equals the transport current: $I_i = I_t$ (series arrangement).

A. Magnetic Flux Density Conforming Formulation

The magnetodynamic a - v formulation consists in finding the gauged magnetic vector potential $\mathbf{a}$ and the scalar electric potential v such that (see e.g. [6])

$$(\nu \mathbf{curl} \mathbf{a}, \mathbf{curl} \mathbf{a}')_{\Omega} + (\sigma \partial_t \mathbf{a}, \mathbf{a}')_{\Omega_c} + (\sigma \mathbf{grad} v, \mathbf{a}')_{\Omega_c} = 0 \quad (1)$$

$$(\sigma \partial_t \mathbf{a}, \mathbf{grad} v')_{\Omega_c} + (\sigma \mathbf{grad} v, \mathbf{grad} v')_{\Omega_c} + \sum_{i=1}^{N_c} I_t V'_i = 0 \quad (2)$$

holds $\forall \mathbf{a}', v', V'_i$, with $\mathbf{grad} v = V_i \mathbf{grad} v_{s,i}$ and $\mathbf{grad} v_{s,i}$ denoting global basis functions [2], or *winding functions* [7].

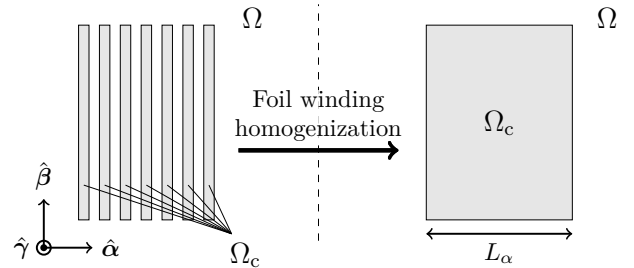


Fig. 1. Foil winding homogenization: N_c resolved conductors (left) replaced by a single homogenized bulk of thickness L_α (right).

As shown in Fig. 1, the foil winding ansatz [1], [2], [4] replaces the successive turns Ω_{c_i} of the coil by a single homogenized bulk. Consequently, the gradient of the electric scalar potential is extended to a continuum:

$$\mathbf{grad} v = \Phi(\alpha) \mathbf{grad} v_s = \sum_{k=1}^{N_b} u_k p_k(\alpha) \mathbf{grad} v_s, \quad (3)$$

in which the N_b spatially dependent global shape functions $p_k(\alpha)$ are not necessarily mesh related. The weak imposition of the current (2) becomes

$$(\sigma \partial_t \mathbf{a}, \mathbf{grad} v')_{\Omega_c} + (\sigma \mathbf{grad} v, \mathbf{grad} v')_{\Omega_c} + \frac{N_c}{L_\alpha} \int_{L_\alpha} I_t \Phi' d\alpha = 0, \quad (4)$$

where $\mathbf{grad} v' = \Phi' \mathbf{grad} v_s$ [2]. Note that (4) enforces the current conservation in every (virtual) elementary foil inside Ω_c , while allowing skin effect along the width (β -axis) of the foils [2]. Recently, this a - v foil winding model has been extended to the j - a - v formulation [5].

B. Magnetic Field Conforming Formulations

The classical full h formulation, relying on winding functions [7] for the weak imposition of the current, consists in finding the magnetic field $\mathbf{h}$ and v such that

$$(\mu \partial_t \mathbf{h}, \mathbf{h}')_{\Omega} + (\rho \mathbf{curl} \mathbf{h}, \mathbf{curl} \mathbf{h}')_{\Omega} + (\mathbf{grad} v, \mathbf{curl} \mathbf{h}')_{\Omega_c} = 0 \quad (5)$$

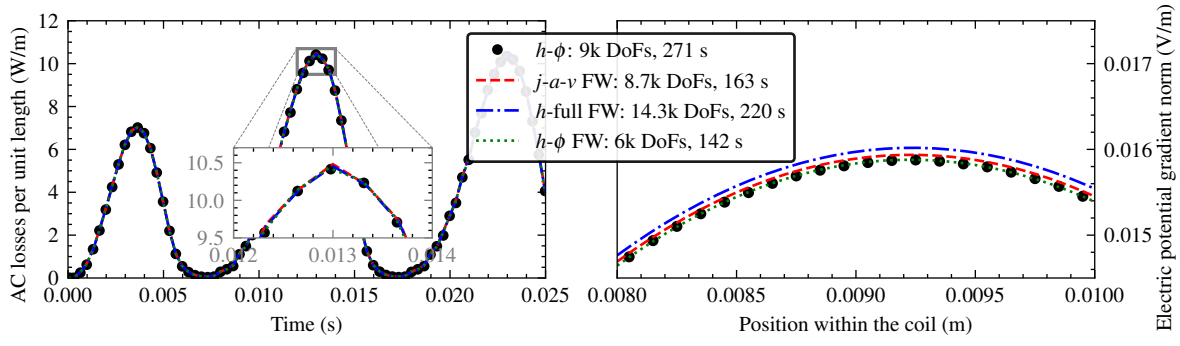


Fig. 2. Evolution of AC losses per unit length in the high-temperature superconducting coil against time (left) and $\mathbf{grad} v$ distribution at the first current peak (right) obtained with the different foil winding (FW) formulations, compared to results with the resolved $h-\phi$ formulation [6]. The legend also gathers the corresponding numbers of degrees of freedom (DoFs), and CPU times.

$$(\mathbf{curl} \mathbf{h}, \mathbf{grad} v')_{\Omega_c} - \sum_{i=1}^{N_c} I_t V'_i = 0 \quad (6)$$

holds $\forall \mathbf{h}', v', V'_i$.

By analogy with the $a-v$ formulation and considering the foil winding ansatz (3), we propose to replace (6) by

$$(\mathbf{curl} \mathbf{h}, \mathbf{grad} v')_{\Omega_c} - \frac{N_c}{L_\alpha} \int_{L_\alpha} I_t \Phi' d\alpha = 0. \quad (7)$$

The combination of (3), (5) and (7) leads to the *full h foil winding formulation*. At the discrete level, $\mathbf{h}$ is discretized using edge shape functions, exclusively.

To avoid the definition of a spurious finite resistivity in non-conducting regions and to reduce the number of required degrees of freedom, we extend the proposed model to the $h-\phi$ formulation. In the non-conducting domain $\Omega_c^C = \Omega \setminus \Omega_c$, $\mathbf{curl} \mathbf{h} = \mathbf{0}$ such that $\mathbf{h} = -\mathbf{grad} \phi$, with ϕ the magnetic scalar potential. Similarly to the classical $h-\phi$ formulation [6], we discretize $\mathbf{h}$ as

$$\mathbf{h} = \sum_{e \in \mathcal{E}(\Omega_c \setminus \partial\Omega_c)} h_e \boldsymbol{\psi}_e + \sum_{n \in \mathcal{N}(\Omega_c^C)} \phi_n \mathbf{grad} \psi_n + I_f \mathbf{c}_f \quad (8)$$

with $\boldsymbol{\psi}_e$ (resp. ψ_n) edge (resp. nodal) shape functions and $\mathbf{c}_f$ a single cohomology basis function associated to the homogenized bulk Ω_c . Instead of imposing the current constraint by strongly fixing the associated global degree of freedom I_f , we still enforce the current weakly using (7). This degree of freedom will be equal, via (7), to the total transport current through the coil cross-section: $I_f = N_c I_t$. The combination of (3), (5) and (7), together with (8), leads to the *$h-\phi$ foil winding formulation*.

III. APPLICATION TO THE COMPUTATION OF AC LOSSES IN AN INSULATED HTS COIL

The proposed formulations have been implemented with the GetDP software. For validation purposes, we consider the 2D simulation of an insulated high-temperature superconducting coil made of 20 turns, which has been described in [5]. In Fig. 2, we compare numerical results, obtained with a third order global polynomial ($N_b = 4$) for the $\Phi(\alpha)$ foil winding discretization, to the $j-a-v$ foil winding model [5] and to the resolved $h-\phi$ formulation [6]. As highlighted, both the AC losses and the $\mathbf{grad} v$ distribution

are accurately reproduced with the two h -conforming foil winding formulations. Furthermore, the $h-\phi$ foil winding formulation allows a significant reduction of the number of degrees of freedom, leading to a faster simulation. This application to a strongly non-linear time-dependent problem highlights the robustness of the presented models.

IV. PERSPECTIVES

The authors would like to emphasize that the introduced models are valid in both 2D and 3D, in both the frequency- and time-domain, as well as for both normal conducting and superconducting applications. In the full paper, normal conducting applications will be discussed and the proposed models will be compared to the (j)- $a-v$ foil winding formulation in 2D and 3D geometries.

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