

Homogenization of Superconducting Strands and Cables for Magneto-Thermal Transient Modelling of Superconducting Magnets

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Simulating magneto-thermal transients in superconducting magnets is important for the design of quench protection systems, but is challenging and computationally expensive due to the multi-scale structure of these magnets and the strongly nonlinear behavior of superconducting materials. In this contribution, we describe a general homogenization workflow allowing to reproduce the macroscopic effects of the small-scale field and current dynamics in superconducting strands and cables without a need to compute them explicitly in the magnet model. This leads to a substantial reduction of the computational effort compared to conventional fully detailed models. The approach relies on simulations at the conductor level (strand or cable), whose results are used to define equivalent homogenized model parameters describing magnetization, loss and inductance. These parameters include hysteretic components inspired by state-of-the-art vector hysteresis models for ferromagnetic materials as well as a novel formulation for interstrand coupling currents.

Index Terms—Type-II superconductors, reduced order modelling, homogenization, finite element analysis.

I. INTRODUCTION

ACCURATELY modelling magneto-thermal transients in superconducting magnets is challenging, mainly due to the need to represent in detail their multi-scale structures. For example, the multi-turn coils of superconducting accelerator magnets are typically wound from a multi-strand cable arranged around the beam pipe, and every strand within the cable contains a high number of twisted superconducting filaments. Inter-filament and inter-strand coupling currents occur in the strands and cables subject to a time-varying field and induce AC loss [1]. This loss, together with the hysteresis and eddy current loss, reduces the temperature margin and can possibly lead to a thermal runaway [2]. In addition, there is an increased interest in quench protection methods that rely on inducing a high current and/or field change rate in a magnet to quickly transition a large part of it to a normal state [2], [3].

Fully detailed magnet models, however, lead to intractable problems. Reduced order models can be developed to lower the simulation effort to practical levels. In that context, we propose a multi-scale homogenization approach, and we apply it to a magnet made of Rutherford cables. At the magnet level, the conductors are represented as homogenized materials, defined based on strand and cable levels simulations performed by efficient two-dimensional models [4], [5].

A particular feature of superconductors is their hysteretic response to external field and transport current due to magnetic flux pinning in the material. As a result, conventional homogenization methods [6] cannot be used directly, but adequate reduced order models must be defined instead. We propose hysteretic models [7] that are designed to account for hysteresis, eddy, and coupling effects in strands and cables.

II. HOMOGENIZATION PROCEDURE

The multi-scale structure of a typical superconducting magnet is illustrated in Fig. 1. The proposed homogenization procedure consists of several steps, further described in the

following subsections: (A) solving detailed strand models [4] to get their transient response in relevant ranges of excitation amplitude and frequency, (B) defining reduced order hysteretic models [7] to reproduce this transient response without a need to solve for the current density distribution, (C) including the reduced order hysteretic models as constitutive laws for homogenized strands in detailed cable models [5] and solving them to get an in-depth description of their transient response, and, finally, (D) reproducing the results with a homogenized cable model, in order to perform transient magneto-thermal simulations of full size magnets [8].

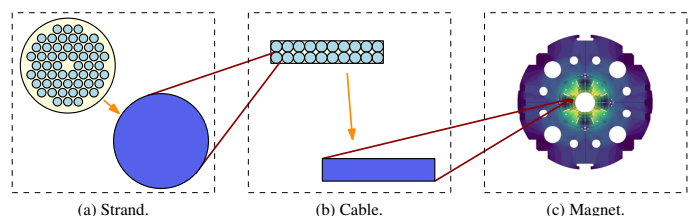


Fig. 1. Illustration of the multi-scale structure of a superconducting accelerator magnet, and of the homogenization workflow proposed in this contribution: from (a) the strand, to (b) the cable, and finally to (c) the magnet [8]. Orange arrows represent homogenization steps.

A. Detailed strand model

Strands are made of superconducting filaments, e.g., Nb-Ti or Nb₃Sn, twisted and embedded in a conducting matrix, e.g., Cu. As a result of this composite structure, different types of current and loss dynamics take place in the strands. Filaments are coupled via currents flowing in the conducting matrix [1] when subject to a time-varying field. Due to flux pinning, superconducting filaments also generate hysteresis loss under transient excitations. Finally, eddy currents take place in the normal conducting matrix and are not negligible for high frequency excitations (typically $\gtrsim 1$ kHz). To account for the interplay between all these effects, we use the CATI

method proposed in [4]. It provides reliable solutions with a lower computational cost than standard 3D strand models. This allows to perform detailed parameter sweeps in a practical amount of time.

B. ROHM and ROHF models

The next step consists in describing the global response of the strands with homogenized material properties [6]. We describe the loss and magnetization induced by an external magnetic field with the reduced order hysteretic magnetization (ROHM) model proposed in [7]. This model is inspired from the energy-based vector hysteresis model for ferromagnetic materials [9], and adapted to composite superconductors to include rate-dependent effects such as coupling and eddy currents. Examples of magnetization curves produced by the ROHM model are shown in Fig. 2.

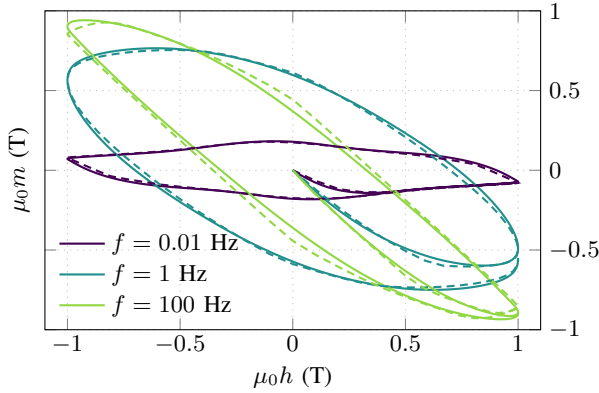


Fig. 2. Average strand magnetization ($\mu_0 m$) for an applied field ($\mu_0 h$) varying sinusoidally with an amplitude of 1 T and frequency f . Solid curves are reference results from [4]. Dashed curves are results from the ROHM model.

The strand response to a time-varying transport current can be described via the magnetic flux internal to the strand, whose behavior is also hysteretic and rate-dependent. To this end, we use a reduced order hysteretic flux (ROHF) model analogous to the ROHM model.

Parameters of the ROHM and ROHF models are fitted based on reference solutions obtained in the previous step. Another contribution at the conference is dedicated to presenting the ROHM and ROHF models in detail [10].

C. Cable model with homogenized strands

Rutherford cables are made of a few tens of strands which are compressed together and transposed to form a periodic structure. Electrical contacts between strands add another coupling mechanism (known as interstrand coupling) which affects the cable magnetization, inductance and loss when it is subject to transient external fields or currents. We model this effect using the CATI method applied to cables [5], with homogenized strands described by the ROHM and ROHF models defined in the previous step.

D. Homogenized cable and magnet model

The final step consists in reproducing the results of the detailed cable model with a homogenized cable model. Individual strand dynamics is reproduced with scaled versions of

the ROHM and ROHF models whereas interstrand coupling dynamics is described with a novel mixed finite element formulation, involving distributed interstrand coupling currents.

Homogenized cable models can then finally be included in a magnet model [8], in order to perform efficient magneto-thermal transient simulations. Results obtained for a simple dipole configuration are shown in Fig. 3.

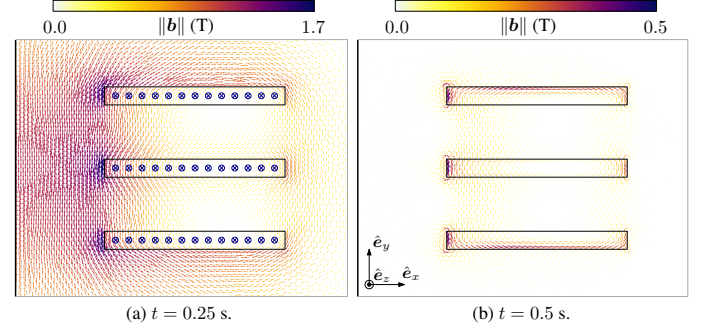


Fig. 3. Application of the reduced order hysteretic models for a simple three-cable dipole magnet with transport current varying sinusoidally in time with frequency of 1 Hz. Arrows represent the magnetic flux density $\mathbf{b}$. Crosses in (a) represent a net transport current. The black line on the left of each subfigure is a symmetry axis. Solution (b) illustrates remnant magnetization at zero current due to hysteresis in the superconducting strand filaments.

III. OUTLOOK AND CONCLUSION

In the final contribution, we will describe the successive steps in detail and provide quantitative analyses of the performance and accuracy of the approach, highlighting its advantages and limitations.

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