

Selection of Spatial Harmonics for Motor Model Order Reduction Using Time-harmonic Eddy-current Analysis

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A motor model order reduction (MOR) based on the decomposition into stator and rotor domains requires a large number of spatial harmonic components of the airgap field to connect the two domains accurately. This article proposes a simple method for selecting dominant spatial harmonics at the airgap, where a time-harmonic eddy-current analysis of the induction motor is carried out before constructing stator and rotor reduced systems. Simulation results show that the proposed method gives an efficient and accurate reduced model of the squirrel cage induction motor.

Index Terms— induction motor, airgap harmonics, model order reduction, nonlinear

I. INTRODUCTION

THE DEVELOPMENT of motor system often requires a large number of trial simulations handling a variety of operating conditions. To avoid large computation cost, an efficient model order reduction (MOR) [1]–[3] is highly demanded for motor simulation. However, the construction of reduced model of induction motor is not very easy because it has to represent the motional electromotive force under variable rotor speed. The domain decomposition approach based on the multiport Cauer ladder network (CLN) method [3]–[5] is one of the promising methods to simulate motor responses under a wide range of operating conditions without using snapshots of the motor operations. The stator and rotor domains are connected by the airgap field with the coordinate transformation between the two domains, which implements a motional electromotive force.

To connect the two domains efficiently and accurately, one should extract dominant spatial harmonic components, which are not known a-priori. Ref. [3] chose dominant harmonics by conducting trial time-dependent analyses, which needs some computational cost. Ref. [4] generated reduced bases of harmonics using the Arnoldi process, which may be cumbersome. This article proposes a simple method for the efficient selection of dominant spatial harmonic components of the airgap field, where only one time-harmonic eddy-current analysis of the induction motor is carried out before constructing the stator and rotor reduced systems.

II. MOR OF INDUCTION MOTOR

A. Domain Decomposition Approach

The stator domain is separated from the rotor domain at the airgap where the two domains are connected by spatial harmonic components of the airgap field. The reduced stator model is generated by the multiport CLN method independently from the reduced rotor model. Their network representations are shown in Fig. 1. In the stator CLN [Fig. 1(a)], I_s and V_s denote the 3-phase source current and voltage vectors, $\mathbf{I}$ and $\mathbf{V}$

are the spatial harmonic components of the magnetic field H_θ and electric field E_z at the airgap interface, and $\tilde{\mathbf{L}}$ is the inductance matrix. For example, H_θ is expanded as

$$H_\theta(\theta, t) = \sqrt{2} \sum_{k=1}^K [H_{ck}(t) \cos(kp\theta) + H_{sk}(t) \sin(kp\theta)] \quad (1)$$

where p is the number of pole pairs and K is the highest order of harmonics considered. Aligning the expansion coefficients, $\mathbf{I}$ is given as

$$\mathbf{I} = [H_{c1}, H_{s1}, \dots, H_{cK}, H_{sK}]^T. \quad (2)$$

Similarly, $\mathbf{V}$ is given by the expansion coefficients of E_z .

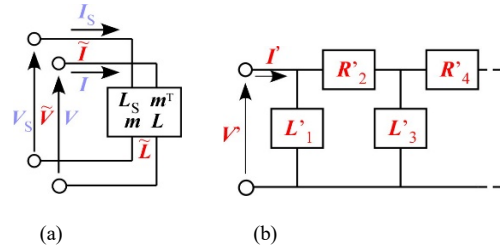


Fig. 1. Multiport representation of (a) stator and (b) rotor domains, where $\tilde{\mathbf{I}} = [I_s, \mathbf{I}]^T$, $\tilde{\mathbf{V}} = [V_s, \mathbf{V}]^T = \tilde{\mathbf{L}} d\tilde{\mathbf{I}}/dt$, and $\mathbf{m}$ is the mutual inductance matrix.

In the rotor CLN [Fig. 1(b)], $\mathbf{I}'$ and $\mathbf{V}'$ are the spatial harmonic components at the airgap and $\mathbf{L}'_{2n-1}$ and $\mathbf{R}'_{2n}$ are inductance and resistance matrices. The boundary conditions at the interface are derived from $\mathbf{H} = \mathbf{H}'$ and $\int \mathbf{E} dt = \int \mathbf{E}' dt'$ where the variables on the left- and right-hand sides are those on the stator and rotor sides, respectively. The interface condition is represented as

$$\mathbf{I}' = \mathbf{T} \mathbf{I}, \quad \Phi' = \mathbf{T} \Phi \quad (3)$$

where ω_m is the mechanical angular velocity, $\mathbf{V} = -d\Phi/dt$, $\mathbf{V}' = -d\Phi'/dt$ and $\mathbf{T}$ is the rotation matrix corresponding to the variable transformation $\theta' = \theta - \int \omega_m dt$.

B. Selection of Airgap Harmonics

Some of the spatial harmonics $k = 1$ to K in (2) are small and not always necessary. To find dominant spatial harmonics at the

airgap, Ref. [3] conducted trial time-dependent analyses, whereas Ref. [4] performed the Arnoldi process generating reduced bases of harmonics. In this study, to simplify the selection procedure for spatial harmonic components, a time-harmonic eddy-current analysis is carried out preparatively to the reduced model construction. For simplicity, only one analysis under the lock condition ($s = 1$) is carried out, because we assume that dominant harmonics are mainly due to the motor geometry and do not greatly depend on the operating condition.

III. NONLINEAR MOR OF INDUCTION MOTOR

A squirrel cage induction motor analyzed in [4] is simulated, which has 2 pole pairs, 24 stator slots, and 34 rotor bars. Figs. 2(a) and (b) show the transient torque waveforms with the slip of 0.15 at 50 Hz under the 3-phase sinusoidal current excitation, which are computed using (0) the FE analysis, (1) the MOR with 1st to 20th ($\times 2$) harmonics without selection, and (2) the MOR with 20×2 harmonics selected by the proposed method. The highest harmonic components are 20 and 69 in the cases of (1) and (2). For comparison, Fig. 2(c) shows the torque waveform computed using (3) the MOR with 40 reduced bases generated by the Arnoldi process [4]. The RMS errors compared with the FE analysis are indicated in Fig. 2. The computed average torques are (0) 2.68, (1) 2.81, (2) 2.71, and (3) 2.35 Nm, respectively. The MOR (1) without harmonics selection fails to reconstruct the torque ripple. The MOR (3) with harmonics reduction by the Arnoldi process roughly represents the torque ripple but fails to predict the average torque. According to [4], the reduced model by the Arnoldi process requires 64 bases for accurate modeling of this motor. In contrast, the proposed selection method (2) accurately reconstructs the torque even with 20×2 harmonics. Fig. 3 shows the speed-dependence of average torque, where the proposed MOR achieves an accurate reconstruction of torque property.

By using the parameterized CLN method, a nonlinear motor MOR is realized [5]. Assuming that dominant harmonics are mainly due to the motor geometry, the dominant harmonic components are selected based on the linear time-harmonic analysis with the initial permeability. Fig. 4 shows the torque waveform with the magnetic saturation, where the proposed method reconstructs the waveform accurately.

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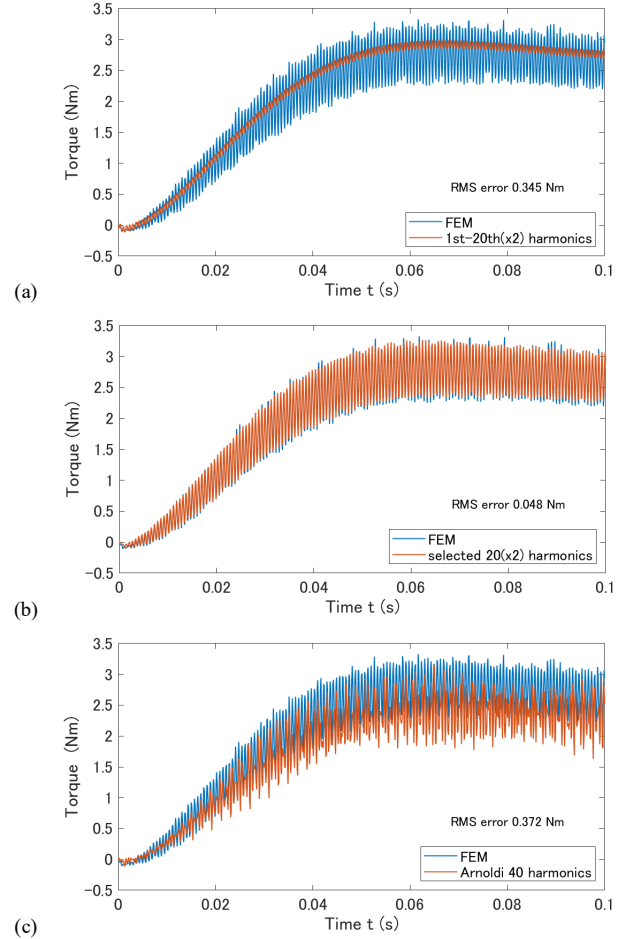


Fig. 2. Torque waveforms: (a) with 1st to 20th ($\times 2$) harmonics without selection (1), (b) with 20×2 harmonics selected by the proposed method (2), and (c) 40 bases reduced by the Arnoldi process (3).

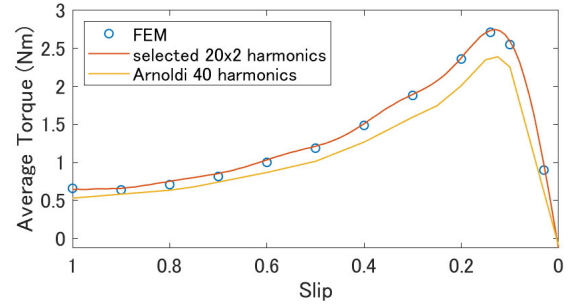


Fig. 3. Speed dependence of average torque.

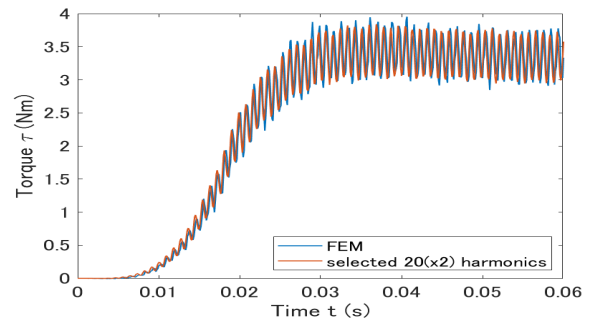


Fig. 4. Torque waveform with nonlinear property.