

Scalable Uncertainty Quantification Framework for High-Frequency Electromagnetic Wave Analysis Using Non-Intrusive Polynomial Chaos and Parallel Finite Element Methods

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This study proposes a scalable framework for uncertainty quantification (UQ) in large-scale high-frequency electromagnetic wave problems by integrating Non-Intrusive Polynomial Chaos (NIPC) methods with parallel finite element solvers. The approach addresses the critical need to model uncertainties in material properties, geometric configurations, and boundary conditions, which significantly impact performance in advanced electronic and communication systems. Key innovations include the use of Karhunen-Loeve Expansion (KLE) to reduce the dimensionality of random fields and regression-based NIPC for efficient probabilistic analysis. Computational optimizations such as coarse-mesh eigenvalue analysis and hierarchical domain decomposition further enhance scalability and efficiency, enabling the framework to handle large-scale simulations effectively. Numerical experiments on the TEAM Workshop Problem 29 model validated the method's ability to accurately quantify uncertainties while significantly reducing computational costs compared to traditional Monte Carlo approaches. The results demonstrated precise modeling of spatially correlated uncertainties and their influence on electromagnetic field distributions. This work bridges the gap between deterministic solvers and advanced UQ methods, providing a robust and computationally efficient tool for real-world applications in electromagnetic compatibility analysis and the design of next-generation electronic devices.

Index Terms—Uncertainty Quantification, Non-Intrusive Polynomial Chaos, High-Frequency Electromagnetic Waves, Finite Element Method

I. INTRODUCTION

Uncertainty Quantification (UQ) [1] is essential in computational engineering, particularly in high-frequency electromagnetic wave problems. Such problems often arise in advanced electronics and communication technologies, where uncertainties in material properties, geometric configurations, and boundary conditions can significantly affect system performance. These uncertainties must be quantified to ensure the reliability and functionality of designs.

Monte Carlo (MC) methods [2] have traditionally been used for UQ due to their simplicity and general applicability. However, their computational inefficiency, particularly for large-scale problems, limits their practical use. Achieving accurate results often requires a prohibitively large number of simulations, rendering MC methods unsuitable for problems with high-dimensional input spaces or complex geometries. In contrast, Ghanem and Spanos proposed a non-statistical method, i.e. the Spectral Stochastic Finite Element Method (SS-FEM) [3]. This method is known for its extremely high computational efficiency, since it performs spectral-expands both input and output fields including uncertainties in a probability space and can evaluate the probability distribution of the outputs without sampling. However, this method requires large modifications to a deterministic simulation code. Such code modification is not an easy task, especially for highly complex programs such as large-scale parallel analysis solvers.

Non-Intrusive Polynomial Chaos (NIPC) methods [4] address these limitations by combining the efficiency of Polynomial Chaos (PC) expansions with the flexibility of

treating deterministic solvers as black boxes. Unlike intrusive methods, NIPC does not require modifications to existing simulation codes, making it a practical choice for large-scale applications. The method models uncertainties as random fields and reduces their dimensionality using techniques such as Karhunen-Loeve Expansion (KLE), enabling efficient and scalable uncertainty analysis.

This study integrates NIPC with a parallel finite element solver designed for high-frequency electromagnetic wave problems. The finite element method (FEM) is well-suited for such analyses due to its ability to handle complex geometries and localized mesh refinements. By maintaining the solver's original structure, the proposed framework capitalizes on FEM's robustness and scalability while incorporating UQ capabilities.

To further enhance computational efficiency, this work introduces optimizations such as coarse-mesh eigenvalue analysis and hierarchical domain decomposition. These techniques reduce the computational burden associated with preprocessing and enable efficient parallelization, ensuring scalability for large-scale simulations.

The effectiveness of the proposed framework is demonstrated through numerical experiments on the TEAM Workshop Problem 29 cavity resonator model [5]. The results show that the integrated approach achieves accurate uncertainty quantification with significantly reduced computational costs, making it a viable tool for applications such as electromagnetic compatibility analysis and advanced device design.

By bridging the gap between computational efficiency and high-fidelity uncertainty representation, this framework

provides a robust and scalable solution for addressing real-world challenges in high-frequency electromagnetic wave analysis.

II. RESULTS AND DISCUSSIONS

The proposed framework's effectiveness was evaluated using the TEAM Workshop Problem 29 cavity resonator model, shown in Fig. 1, a benchmark model in electromagnetic analysis. Numerical experiments demonstrated that the integrated NIPC and FEM approach achieved significant computational efficiency without sacrificing accuracy.

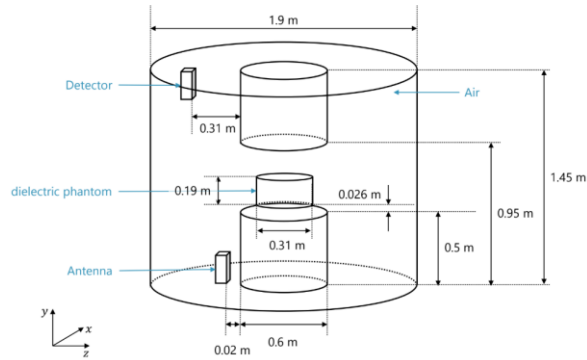


Fig. 1. Team Workshop Problem 29 cavity resonator model, commonly used as a benchmark for electromagnetic field analysis, illustrating key components such as the antenna, detector, and dielectric phantom.

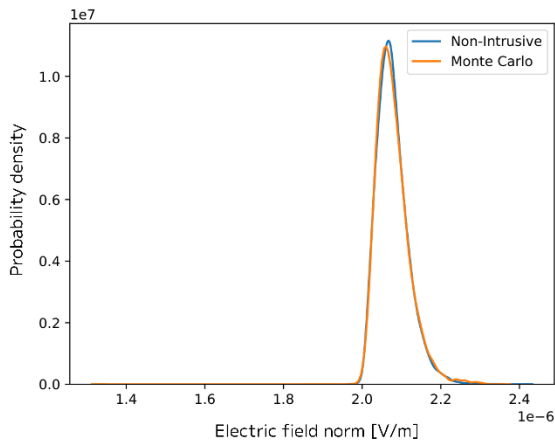


Fig. 2. Comparison of Probability Density Functions Probability density functions of the electric field norm at a specific point, obtained using NIPC and MC simulations.

One of the key results was the precise quantification of uncertainty in electromagnetic field distributions. The probabilistic distributions of output parameters, such as electric field magnitudes at specific points, closely matched those obtained through MC simulations but required significantly fewer computational resources. As shown in Fig. 2, the probability density function of the electric field norm demonstrates that the NIPC method closely aligns with the results from Monte Carlo simulations, validating the accuracy of the proposed method. In terms of computational cost, the NIPC method required only 145 seconds, whereas the Monte Carlo method took 20,605 seconds, illustrating a significant

reduction in computational time while maintaining comparable accuracy.

Computational efficiency was further enhanced by employing coarse-mesh eigenvalue analysis during the preprocessing phase. This approach reduced the computational complexity associated with solving large eigenvalue problems, which is a common bottleneck in high-fidelity simulations. Additionally, hierarchical domain decomposition enabled efficient parallelization, allowing the framework to scale effectively for large-scale simulations.

The results also highlighted the versatility of the framework in handling complex geometries and varying input uncertainties. By modeling input uncertainties as random fields using KLE, the framework captured the spatial correlations of uncertainties, leading to more realistic and robust simulations. This capability is particularly valuable for analyzing advanced electronic devices and communication systems, where localized variations in material properties can significantly impact performance.

III. CONCLUSION

This study developed an efficient framework integrating NIPC methods with a parallel finite element solver for UQ in large-scale high-frequency electromagnetic wave problems. By leveraging KLE and computational optimizations such as coarse-mesh eigenvalue analysis and hierarchical domain decomposition, the proposed method achieves high accuracy and computational efficiency.

Numerical experiments using the TEAM Workshop Problem 29 model validated the framework's ability to accurately quantify uncertainties while significantly reducing computational costs. The results demonstrate the framework's potential for broad application in advanced electromagnetic analysis, including the design of electronic devices and evaluation of electromagnetic compatibility.

In conclusion, this work bridges the gap between computational feasibility and the need for high-fidelity UQ in complex engineering systems. It provides a robust and scalable solution for addressing real-world challenges, making it a valuable tool for future advancements in electromagnetic research and engineering.

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