

Fast electromagnetic simulation of meta-surfaces

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In this paper a new electromagnetic solver, specifically tailored for scattering by meta-surfaces, is presented. The solver is based on a source integral volume formulation and implements a novel QR-compression approach that exploits the specific geometric structure of the meta-surface. Together with a specifically designed pre-conditioner, the proposed solver shows the capability to handle a large number of variables.

***Index Terms*—Full wave analysis, integral equations, QR decomposition, meta-surfaces.**

I. INTRODUCTION

The fast and accurate simulation of electromagnetic scattering by meta-surfaces [1] is a topic of definite interest in the current literature, given the increasing use of meta-surfaces in a variety of different contexts. This is the case in medical devices [2] or mobile electronics [3], where the demand for miniaturization has made classical optical lenses no longer suitable. The superior performance of meta-surfaces is ensured by their excellent abilities to manipulate light and electromagnetic radiation and the possibility of being engineered in order to match the desired phase profile, phase retardation and so on [4].

From a very general perspective, a meta-surface is constituted by a large number of geometrically separated elementary scatterers, usually termed as meta-atoms. Their shape, dimension, orientation, and arrangement affect the overall electromagnetic response of the array. The numerical simulation of a realistic meta-surfaces constitutes a large scale problem, where the number of unknowns can easily be huge. In addition to the analysis of meta-surfaces, electromagnetic solvers play a key role in their design, where iterative optimization methods require the solution of (at least) one directed problem at each step [5], [6].

In this framework, the computational cost is the major issue, especially for facing the design problem where different (many) configuration need to be simulated. Integral formulations are an effective choice since they require to discretize only the regions occupied by meta-atoms. In particular, in this work is adopted a Volume Integral Formulation (VIE) [7], where a loop and star basis is chosen to effectively represent the unknowns.

This formulation has been proven to be particularly effective in a large variety of applications ranging from static problems to microwaves. In this paper, the potential of this approach is extended to meta-surfaces yielding large scale problems. To achieve this goal, a proper pre-conditioner and a proper matrix-by-product compression obtained via a QR-recursive scheme is introduced.

This contribution is organized as follows: in Section II VIE formulation is briefly summarized, in Section III the compression method and the pre-conditioner are presented

while in Section IV some preliminary numerical examples are reported. Finally, conclusions are drawn in Section V.

II. MATHEMATICAL FOUNDATION

The proposed method is based on a source integral volume formulation [7] in time-harmonic operations, i.e. the $e^{j\omega t}$ behavior is assumed for electromagnetic fields.

The computational domain is denoted by Ω and it is constituted by multiple connected components $\Omega_i \subseteq \mathbb{R}^3$, with Ω_i Lipschitz domain, that are the elementary meta-atoms forming the meta-surface. The electrical conductivity and permittivity is homogeneous in each Ω_i and the materials are linear, non-spatially dispersive and non-magnetic. Finally, there are no source charges in any of the cavities of Ω .

Under these assumptions, the problem is modeled by the following integral equation

$$\begin{aligned} & [\sigma(\mathbf{r}) + j\omega\chi(\mathbf{r})\varepsilon_0]^{-1} \mathbf{J}(\mathbf{r}) + j\omega\mu_0 \int_{\Omega} \mathbf{J}(\mathbf{r}') g(\mathbf{r} - \mathbf{r}') dV' \\ & + \frac{1}{j\omega\varepsilon_0} \nabla \int_{\partial\Omega} (\mathbf{J} \cdot \hat{\mathbf{n}})(\mathbf{r}') g(\mathbf{r} - \mathbf{r}') dS' = \mathbf{E}_0(\mathbf{r}), \text{ in } \Omega, \end{aligned} \quad (1)$$

where $\mathbf{J}(\mathbf{r}) = [\sigma(\mathbf{r}) + j\omega\varepsilon_0\chi(\mathbf{r})] \mathbf{E}(\mathbf{r})$ is the sum of the conduction and the polarization current density, σ is the electrical conductivity, χ is the dielectric susceptibility, ε_0 is the dielectric permittivity of the free-space, $g(\mathbf{r}) = e^{-jkr}/(4\pi r)$ is the scalar free space Green's function, $k = \omega\sqrt{\varepsilon_0\mu_0}$ is the free-space wavelength, and $\mathbf{E}_0$ is the applied electric field.

The weak form of Equation (1) is discretized by means of the Galerkin method. The unknown $\mathbf{J}$ is expanded on the the loop-tree basis, i.e. as a combination of loop and star functions as described in [7], [8], leading to

$$\mathbf{Z} \mathbf{I} = \mathbf{v}_0, \quad (2)$$

where $\mathbf{Z}$ is a fully populated matrix.

III. PRECONDITIONING AND COMPRESSION

The proposed method is meant for large scale problems, i.e. meta-surfaces constitutes by thousands of meta-atoms. In this setting, Equation (2) cannot be solved by direct methods and

iterative solvers must be employed to drastically reduce the computational time.

The aim of an iterative method is to repeatedly compute the $\mathbf{Z}\mathbf{I}_n$ or $\mathbf{Z}^H\mathbf{I}_n$ products, until the residual of Equation (2) is below a proper threshold. Hence, it is mandatory to minimize: (i) the computational cost for evaluating the product $\mathbf{Z}\mathbf{I}_n$, (ii) the memory needed to store the matrix $\mathbf{Z}$, and (iii) the number of required iterations.

A QR based recursive compression method is adopted to deal with problems (i)-(ii). In a nutshell, this compression method splits the product $\mathbf{Z}\mathbf{I}_n$ into a near and a far part based on a multi-level grid scheme [10]. The near part is computed exactly, while the far part can be effectively approximated by means of a QR decomposition [9].

The novelty introduced in this work consists in conveniently exploiting the geometrical structure of meta-lens, to increase the performances of the compression method. Specifically, the multi-level grid is designed by a meta-atoms oriented approach, where the near part consists of only the interactions of a meta-atom with itself, and each meta-atom is assigned to only one box of the grid, at any refinement level.

Similarly, The pre-conditioner exploit the structure of the array consisting in an ensemble of geometrically separated meta-atoms. Specifically, let $\mathbf{Z}_{i,j}$ be the block of $\mathbf{Z}$ that accounts for the interaction between the DoFs of the i -th meta-atom and the DoFs of the j -th meta-atom. The pre-conditioner is obtained by inverting the matrix $\mathbf{Z}$ where all the mutual interactions between different meta-atoms have been neglected, i.e. the pre-conditioner is the inverse of the following block-diagonal matrix

$$\text{diag}(\mathbf{Z}_{1,1}, \dots, \mathbf{Z}_{N,N}).$$

The assembly of the pre-conditioner has a minimal computational cost since it grows only linearly with the number of meta-atoms and reduces to $\mathcal{O}(1)$ if all the meta-atoms are identical.

IV. NUMERICAL RESULTS

The above method has been tested on a meta-lens array with $N = 2000$ meta-atoms. Specifically, a particular array structure is considered, obtained by the replication of the same meta-atoms, which is a sphere. The spheres have a radius $R_s = 100$ nm, while the meta-atoms are arranged in a Vogel's spiral, as discussed in [11].

The discretization of the single sphere is reported in Figure 1(a), while the overall mesh is obtained by the replication of the single spheres. The simulated meta-surface is depicted in Figure 1(b). The discretization of the single sphere consists of 321 points, 256 edges, with 465 loop and 95 star unknowns, i.e. a total of 560 DoFs. It turns out that the total number of DoFs for the array is 1 120 000.

Full characterization of the performance of the method in the presented case is deferred to the full version of this contribution.

V. CONCLUSION

In this paper a novel solver for the computation of the scattering by meta-lens is proposed. This new approach allows

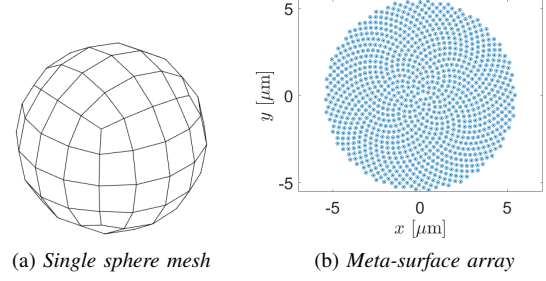


Fig. 1. Left: discretization of the meta-atom. Right: simulated meta-surface.

to treat large scale problems by means of a VIE formulation, with loop and star basis functions. A complete discussion of the performance of the proposed method will be presented in the full paper version.

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