

Analytical solutions for material design

A. Tamburrino^{1,2}, V. Mottola¹, and S. Ventre¹

¹ DIEI, Università degli Studi di Cassino e del Lazio Meridionale, Cassino, 03043 Italy

² Department of Electrical and Computer Engineering, Michigan State University, East Lansing, MI-48824, USA.

The electromagnetic response of devices as dielectric resonator is determined by resonant modes. It turns out that methods capable of properly designing scatterers to achieve prescribed resonance modes at prescribed frequencies are of great interest, enabling precise control of the electromagnetic field by taking advantage of improvements in the fabrication of meta-materials.

In this contribution, a new design method is proposed to compute the dielectric permittivity tensor of a scatterer exhibiting prescribed resonance modes, at prescribed frequencies. The dielectric permittivity is computed via an explicit and analytical formula that, moreover, highlights two relevant results: (i) only two distinct modes can be prescribed at the same resonance frequency, and (ii) the electromagnetic response can be changed at will (tunability) by scaling the dielectric susceptibility by a proper multiplicative constant.

Index Terms—Inverse spectral problem, material design, meta-materials, tunability.

I. INTRODUCTION

Anisotropic materials and materials with continuous spatially varying electromagnetic properties are the key to obtain a variety of different electromagnetic phenomena that can be conveniently exploited in applications. The recent progress in fabrication of meta-materials [1] made it possible to practically implement this kind of materials, leaving the space for the design of devices exhibiting desired electromagnetic behavior. As discussed in [2], [3], anisotropic and inhomogeneous materials offer, among the others, the possibility to scatter incident electromagnetic fields in a prescribed way.

The design of materials with a prescribed electromagnetic response is mainly conducted via iterative optimization methods, which allow to take into account fabrication constraint also [4], [5]. However, these methods suffer from some limitations related to the high computational cost of full-wave electromagnetic simulations.

In this contribution, a different approach is proposed based on the possibility to analytically solve the design problem, giving an explicit formula to obtain the material property realizing the prescribed electromagnetic response.

The problem of interest is the design of anisotropic and inhomogeneous materials exhibiting prescribed resonance modes at prescribed frequencies. Resonance modes play a key role for different devices, such as plasmonic and dielectric nano-resonators [6].

Furthermore, the proposed method allows to design tunable materials in which, by scaling the dielectric permittivity by a multiplicative coefficient, it is possible to change the electromagnetic response, i.e. the resonance mode which is excited at a given frequency.

The analytical solution of the proposed method allows, also, to highlight the concept of *essential modes*: due to physical limitations, at most two different modes can be prescribed at the same resonance frequency.

II. MATHEMATICAL FORMULATION

The aim of the design method is to determine the dielectric permittivity tensor χ of a bounded scatterer placed in free

space, in such a way that it exhibits prescribed resonance modes at prescribed frequencies (see Figure 1). The scatterer

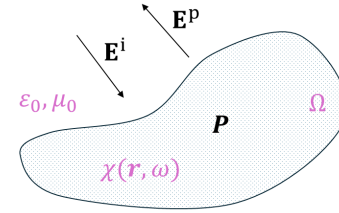


Fig. 1. Bounded dielectric scatterer placed in free-space. The method finds the dielectric permittivity χ to achieve prescribed resonance modes at prescribed frequencies.

occupies the region Ω , a $e^{j\omega t}$ behavior is assumed for the fields and the dielectric is considered linear, anisotropic, inhomogeneous, non spatially dispersive and non-magnetic. Under these assumptions, the constitutive relationship is

$$\mathbf{P}(\mathbf{r}, \omega) = \varepsilon_0 \chi(\mathbf{r}, \omega) \mathbf{E}(\mathbf{r}, \omega), \quad (1)$$

where $\mathbf{P}$ is the polarization density, $\mathbf{E}$ is the electric field and ω is the angular frequency. Consequently, the scattering is governed by the following equation

$$\frac{1}{\varepsilon_0} \chi^{-1} \mathbf{P} - \mathcal{E}(\omega) \mathbf{P} = \mathbf{E}^i \quad (2)$$

The (2) is obtained starting from (1) by splitting the total electric field $\mathbf{E}$ in the sum of incident field $\mathbf{E}^i$ and of $\mathbf{E}^P$, which is the field due to $\mathbf{P}$ and recalling that $\mathbf{E}^P = \mathcal{E}(\omega) \mathbf{P}$, where $\mathcal{E}(\omega)$ is a linear integral operator [7].

A resonance mode is a non-trivial solution $\mathbf{P}$ of the resonance mode equation

$$\mathcal{E}(\omega) \mathbf{P} = \frac{1}{\varepsilon_0} \chi^{-1} \mathbf{P}, \quad (3)$$

obtained by zeroing the incident field term in the governing equation. In this contribution, a generalized form of the (3) is considered

$$\mathcal{E}(\omega) \mathbf{P} = \xi \frac{1}{\varepsilon_0} \chi^{-1} \mathbf{P}, \quad (4)$$

in which the eigenvalue $\xi \in \mathbb{C}$ is added in order to obtain tunable materials, which is one of the goal of the design method.

III. DESIGN METHOD

Given a set of prescribed resonance modes, as triplets $(\omega_k, \xi_k, \mathbf{P}_k)_k$, the design method consists in two distinct phases. As first step a solution χ of equation (4) is computed for each different prescribed frequency and, then, the overall frequency response of the dielectric is obtained by a proper interpolation.

For a fixed frequency ω , the solution of Equation (4) is [8]

$$\chi^{-1}(\mathbf{r}, \omega_k) = \sum_{l,m=1}^2 \Gamma_{l,m}(\mathbf{r}) \frac{[\varepsilon_0 \mathcal{E}(\omega_k) \mathbf{P}_l](\mathbf{r})}{\xi_l} \mathbf{P}_m^*(\mathbf{r}), \quad (5)$$

in which the only unknown quantities are the coefficient $\Gamma_{l,m}$ that can be easily computed.

As it can be seen, the sought dielectric permittivity tensor can be obtained by an explicit and analytical formula, which allows to solve the design problem without the need of any optimization scheme.

The tunability of the designed material is, instead, ensured by the presence of the eigenvalue ξ . The key observation is that if ξ_k , $\mathbf{P}_k$ and χ are solution of the generalized problem (4), then $\xi_k^{-1}\chi$ and $\mathbf{P}_k$ are solution of the resonance mode equation (3). For example, if the design is conducted for two triplets $(\omega_1, \xi_1 = 1, \mathbf{P}_1)$ and $(\omega_2, \xi_2 = 10, \mathbf{P}_2)$, then the material χ resonates at the frequency ω_1 on the mode $\mathbf{P}_1$, while the material $\chi/10$ resonates at the frequency ω_2 on the mode $\mathbf{P}_2$. This ensures that the electromagnetic response of the scatterer can be controlled by simply scaling its dielectric permittivity in a controlled manner.

The application of the design method is exemplified in Figure 2. A dielectric cylinder with square cross-section is designed to exhibits three different modes resembling the numbers “0”, “1”, “2” at three different frequencies. The tunability

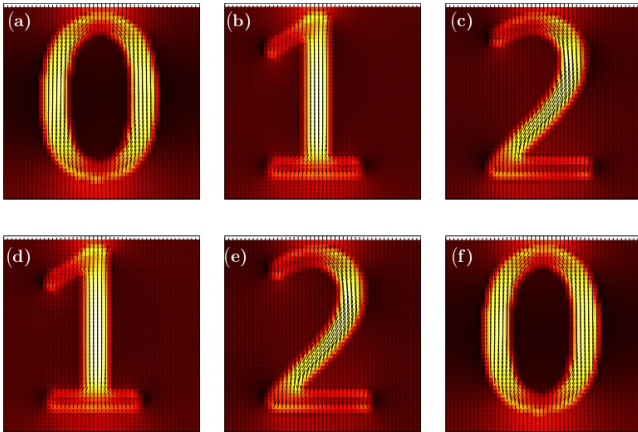


Fig. 2. Polarization density fields induced in a translation invariant cylinder with square cross section. In the first row the permittivity is equal to χ , for $\xi = 1$, while in the second one the permittivity is equal to $\chi/2$, for $\xi = 2$.

is clearly demonstrated. Indeed, the panels (a-c) refer to the electromagnetic behavior of a scatterer characterized by a

dielectric permittivity equal to χ ($\xi = 1$), while the panels (d-f) refer to a scatterer with dielectric permittivity equal to $\chi/2$ ($\xi = 2$).

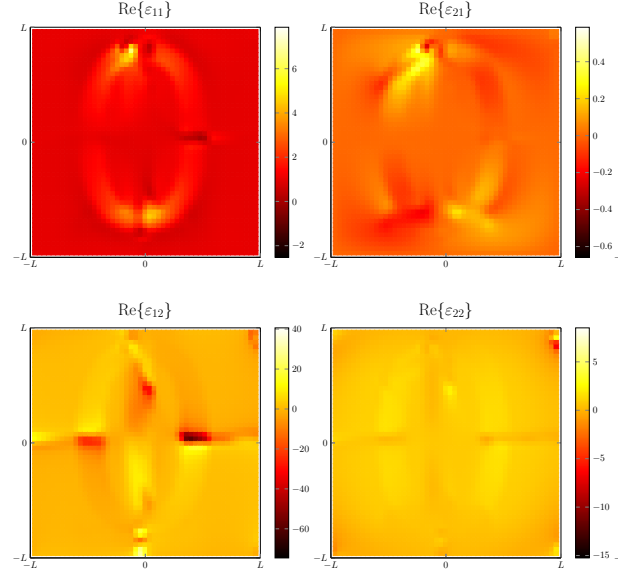


Fig. 3. Real part of the designed χ^{-1} , for the first frequency (first column of Figure 2)

The expression (5) enables to introduce the concept of *essential mode*, also. Due to the physical limitations, it is possible to prove that at most two different modes can be prescribed for a given angular frequency. This is significant because it allows to rigorously define the limitation in the material design.

IV. CONCLUSIONS

In this contribution a new method for the electromagnetic resonance design of dielectric materials is presented. The method gives an explicit formula to obtain the dielectric permittivity tensor of a material exhibiting prescribed resonance modes at prescribed frequencies. Furthermore, the methods allow the synthesis of tunable materials and it clearly demonstrates that at most two different modes can be prescribed for a prescribed frequency.

REFERENCES

- [1] N. Engheta and R. W. Ziolkowski, *Metamaterials: Physics and Engineering Explorations*. John Wiley & Sons, June 2006.
- [2] J. B. Pendry, D. Schurig, and D. R. Smith, “Controlling Electromagnetic Fields,” *Science*, vol. 312, no. 5781, pp. 1780–1782, 2006.
- [3] U. Leonhardt, “Optical Conformal Mapping,” *Science*, vol. 312, no. 5781, pp. 1777–1780, 2006.
- [4] T. W. Hughes, M. Minkov, I. A. D. Williamson, and S. Fan, “Adjoint Method and Inverse Design for Nonlinear Nanophotonic Devices,” *ACS Photonics*, vol. 5, pp. 4781–4787, Dec. 2018. Publisher: American Chemical Society.
- [5] K. Yao, R. Unni, and Y. Zheng, “Intelligent nanophotonics: merging photonics and artificial intelligence at the nanoscale,” *Nanophotonics*, vol. 8, pp. 339–366, Jan. 2019. Publisher: De Gruyter Section: Nanophotonics.
- [6] P. Lalanne, W. Yan, K. Vynck, C. Sauvan, and J.-P. Hugonin, “Light Interaction with Photonic and Plasmonic Resonances,” *Laser & Photonics Reviews*, vol. 12, no. 5, p. 1700113, 2018.
- [7] J. G. Van Bladel, *Electromagnetic fields*, vol. 19. John Wiley & Sons, 2007.
- [8] A. Tamburrino and S. Ventre, “Synthesis of resonant modes in electromagnetics”, submitted for publication.