

Resonance transition radiation in one-dimensional moiré lattices

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When free electrons travel photonic crystals (PCs), multiple transition radiations are generated, yielding highly directional resonance radiation if the emitted electromagnetic (EM) waves are coherent. In this work, we have investigated the resonance transition radiation regulated by moiré lattices, which can offer greater flexibility in manipulating EM waves compared to conventional PCs, facilitating more intricate free-electron radiation phenomena. In particular, an exponential scattering matrix cascading approach is proposed, which can significantly decrease the necessary cascading steps, signifying a substantial reduction in computational resources. The angular energy density distribution is obtained and the velocity forbidden zone is observed. Our work will provide a foundation for exploring the promising wave behavior within moiré lattices.

Index Terms—optical superlattices, electromagnetic radiation, resonance, numerical analysis

I. INTRODUCTION

MOIRÉ patterns, resulting from the interference of two partially distinct periodic patterns, have introduced a novel dimension for developing new conceptual physics systems, leading to numerous anomalous phenomena such as superconductivity in magic-angle graphene [1], second harmonic generation [2], and optical localization [3], to name a few. By manipulating interference parameters, such as the difference in periodic length and twisting angle, the band structure of moiré lattices may be regulated, offering a platform for exploring exotic wave behavior. Recent studies demonstrate that gratings or photonic crystals (PCs) can be utilized to control free-electron radiation [4] and moiré lattices may offer potentials to achieve specific radiation due to their intricate structures. In this work, we have calculated the band structure of the moiré lattice using the transfer matrix method (TMM). In addition, the forward and backward resonance transition radiations [4] produced by free electrons traversing a finite-length moiré lattice have been investigated within the scattering matrix method (SMM). Moreover, an exponential cascading approach has been proposed to decrease the number of cascading steps, reducing the calculation time. Our research establishes a numerical basis for investigating exceptional wave behavior in one-dimensional moiré lattices.

II. THEORY AND METHODS

A. Propagation mode analysis

moiré lattice is usually obtained from the superposition of two one-dimensional PCs with different periods T_1 and T_2 as illustrated in Fig. 1. For example, $\epsilon^M(z) = \epsilon_1$ when $\epsilon^{\text{PC1}}(z) = \epsilon_1$ or $\epsilon^{\text{PC2}}(z) = \epsilon_1$; otherwise, $\epsilon^M(z) = \epsilon_2$, where $\epsilon^{\text{PC1}}(z)$ and $\epsilon^{\text{PC2}}(z)$ are the permittivity distributions of PC1 and PC2, respectively. Here, PC1 and PC2 are both composed of non-magnetic dielectric materials with relative permittivities ϵ_1 and ϵ_2 arranged periodically in an alternating manner, and have the same filling ratio $\eta_n = d_1^{\text{PCn}}/d_2^{\text{PCn}}$, where d_1^{PCn} denotes the thickness of dielectrics l inside PC n ($n = 1, 2$). Therefore, the period of the moiré lattice reads $L = N_1 T_1 = N_2 T_2$.

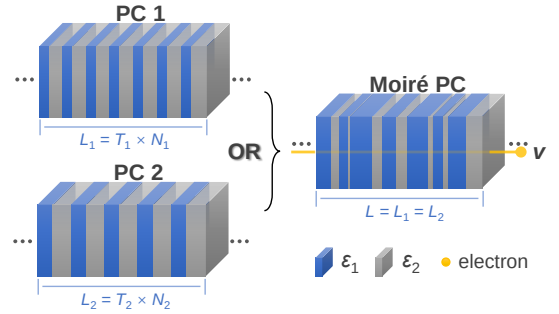


Fig. 1. Schematic of a free electron with the velocity of $\mathbf{v}$ traveling through a moiré lattice. The moiré lattice with a period $L = N_1 T_1 = N_2 T_2$ is obtained by applying the Boolean operator 'OR' between N_1 cells of PC1 with period T_1 and N_2 cells of PC2 with period T_2 .

We consider a free electron with charge q_e and velocity $\mathbf{v} = \hat{z} \cdot \beta c_0$ traveling through a moiré lattice, as shown in Fig. 1, where c_0 denotes the light speed in vacuum. The movements of the electron can be simulated by the current density $\mathbf{J}_e(\mathbf{r}, t) = \hat{z} \cdot \delta(x) \delta(y) \delta(z - \beta c_0 t) q_e \beta c_0$. For convenience, we conduct the analysis in the mixed domain $\{k_x, k_y, z, \omega\}$. As a result, the current density can be rewritten as $\mathbf{J}_e = \hat{z} \cdot q_e e^{i k_e z}$, where $k_e = \omega/(\beta c_0)$ with the help of Fourier transformation [5]. For simplicity, we omit (k_x, k_y, z, ω) in the following derivation. Now, the governing equation for the electric field $\mathbf{E}_n$ in the layer n is written as

$$\mathbb{M}_n \cdot \mathbf{E}_n = i \omega \mu_0 \mathbf{J}_e, \quad (1)$$

where

$$\mathbb{M}_n = \begin{pmatrix} k_y^2 - \partial_z^2 & -k_x k_y & i k_x \cdot \partial_z \\ -k_y k_x & k_x^2 - \partial_z^2 & i k_y \cdot \partial_z \\ \partial_z \cdot i k_x & \partial_z \cdot i k_y & k_x^2 + k_y^2 \end{pmatrix} - \epsilon_n k_0^2 \cdot \mathbb{I}_3. \quad (2)$$

Here, $\mathbb{I}_3$ denotes the identity matrix with the order of 3; ∂_z denotes the partial derivative with respect to z ; μ_0 is the permeability in vacuum; and $k_0 = \omega/c_0$. Solutions of the non-homogeneous derivative equation (1) consists of the general solutions $\mathbf{E}_n^{\text{g.m}}$ and the particular solution $\mathbf{E}_n^{\text{p}} e^{i k_e z}$, which can

be calculated according to $\mathbf{E}_n^{\mathbf{g},m} = \text{NullVector}[\mathbb{M}_n(k_{z,n}^m)]$ and $\mathbf{E}_n^{\mathbf{p}} e^{ik_e z} = (\mathbb{M}_n(k_e))^{-1} \cdot i\omega\mu_0 \mathbf{J}_e$, where $\mathbb{M}_n(k_{z,n}^m)$ is obtained by replacing ∂_z with $k_{z,n}^m$ in $\mathbb{M}_n$ with $k_{z,n}^m$ satisfying the condition $\det[\mathbb{M}_n(k_{z,n}^m)] = 0$ and $\mathbb{M}_n(k_e)$ being obtained by replacing ∂_z with k_e in $\mathbb{M}_n$. Hence, the electric field $\mathbf{E}_n$ in layer n reads as

$$\mathbf{E}_n = \tau_n^1 \mathbf{E}_n^{\mathbf{g},1} e^{ik_{z,n}^1 z} + \tau_n^2 \mathbf{E}_n^{\mathbf{g},2} e^{ik_{z,n}^2 z} + r_n^3 \mathbf{E}_n^{\mathbf{g},3} e^{ik_{z,n}^3 z} + r_n^4 \mathbf{E}_n^{\mathbf{g},4} e^{ik_{z,n}^4 z} + \mathbf{E}_n^{\mathbf{p}} e^{ik_e z}, \quad (3)$$

where $\mathbf{E}_n^{\mathbf{g},1(3)} = (-k_{z,n}^{1(3)}, 0, k_x)^T$, $\mathbf{E}_n^{\mathbf{g},2(4)} = (-k_y, k_x, 0)^T$, $\mathbf{E}_n^{\mathbf{p}} = (i\omega\mu_0 q_e / \epsilon_n k_0^2) \cdot (k_e k_x, k_e k_y, k_e^2 - \epsilon_n k_0^2) / (\epsilon_n k_0^2 - k_e^2 - k_x^2 - k_y^2)$, $k_{z,n}^{1(2)} = (\epsilon_n k_0^2 - k_x^2 - k_y^2)^{1/2}$ and $k_{z,n}^{3(4)} = -(\epsilon_n k_0^2 - k_x^2 - k_y^2)^{1/2}$. Here, the coefficients $\tau_n^{1(2)}$ and $r_n^{3(4)}$ represent the amplitudes of the forward and backward waves, respectively, which may be determined with the boundary conditions. Once $\mathbf{E}_n$ is obtained, the magnetic field $\mathbf{H}_n$ can be determined immediately. Now, the scattering matrix $\mathcal{S}_{i,j}^{\mathbf{L}}$ (the superscript ‘L’ denotes the layer) at the interface between two contiguous layers i, j can be derived.

B. Exponential cascading algorithm

To acquire the comprehensive radiation properties, the scattering matrices of each layer need to be concatenated. However, cascading layer by layer is inefficient, and we introduce an exponential cascading algorithm. We first obtain the total scattering matrices for one sub-cell $\mathcal{S}^{\mathbf{C}}$ (the superscript ‘C’ denotes the cell) by cascading the scattering matrices $\{\mathcal{S}_{i,j}^{\mathbf{L}}\}$ at the interfaces within each sub-cell step by step. Now, a recurrence relation can be obtained as $\mathcal{S}_{2n}^{\mathbf{C}} = \mathcal{S}_n^{\mathbf{C}} \star \mathcal{S}_n^{\mathbf{C}}$, where $\mathcal{S}_n^{\mathbf{C}}$ represents the overall scattering matrix derived from cascading the scattering matrices of n sub-cells and $\star$ denotes the cascading operation. The number of sub-cells N can be expanded into a summation form with powers of two, i.e., $N = \sum_i 2^{\alpha_i}$, where $\alpha_i \in \mathbb{N}$. Thus, the total scattering matrix for the entire moiré lattice can be obtained as

$$\mathcal{S}_N^{\mathbf{C}} = \mathcal{S}_{2^{\alpha_1}}^{\mathbf{C}} \star \mathcal{S}_{2^{\alpha_2}}^{\mathbf{C}} \star \dots \star \mathcal{S}_{2^{\alpha_p}}^{\mathbf{C}}, \quad \alpha_i \in \{\alpha_i | \sum_i 2^{\alpha_i} = N\}. \quad (4)$$

Compared to the successive cascading approach [6], the proposed exponential cascading method reduces the number of cascading steps from $\Omega^{\text{suc}} = N - 1$ to $\Omega^{\text{exp}} = \text{Max}\{\alpha_i\} + \text{Num}\{\alpha_i\} - 1$, where N denotes the number of sub-cells.

III. RESULTS AND DISCUSSIONS

Fig. 2 shows the angular energy density $U(\theta, \phi, \omega)$ of the resonance transition radiation [4], [6] outside moiré lattices. Here, the moiré lattices are constructed by superposing two PCs consisting of two materials with relative permittivity $\epsilon_1 = 10.6$ (GaP) and $\epsilon_2 = 2.1$ (SiO₂), whose fill ratios read $\eta_1 = \eta_2 = 0.3$. In addition, the period of PC1 reads $T_1 = 1.0205\lambda_0$ (where $\lambda_0 = 700$ nm); thus the period of PC2 reads $T_2 = N_1 T_1 / N_2$ (here, $N_1 = 16$ and $N_2 = 15$) and the super-period L of the moiré lattices reads $L = N_1 T_1$. As shown in Fig. 2(a), the band structure of the moiré lattices indicates a relatively flat band in the high frequency region, where $\omega_L = 2\pi c_0 / L$ and $k_L = 2\pi / L$. At the operating frequency $\omega_0 = 10\omega_L$, the iso-frequency contours may be determined by computing the dispersion relation for the moiré lattices $f(k_p, k_e, \omega)$, where

$k_p = (k_x^2 + k_y^2)^{1/2}$, with the constraints that (ω_0, k_e) are situated inside the band structure depicted in Fig. 2. Moreover, when the particle’s velocity is within the forbidden zone, directional resonant transition radiation cannot be observed. As shown in Fig. 2(c,d), the calculated forward and backward angular energy density spectra $U_{f,b}(\theta, \phi, \omega)$ coincide well with the iso-frequency contour, indicating the accuracy of the numerical algorithm. The forward and backward radiation, while aligned in direction, differ in intensity: the red branch of the iso-frequency contour in Fig. 2(b) displays enhanced forward radiation, whilst the blue and yellow branches indicate more backward radiation. Hence, by distinguishing the relative intensity of forward and backward radiation, one can estimate the approximate range of the particle’s velocity, facilitating later identification of its exact velocity.

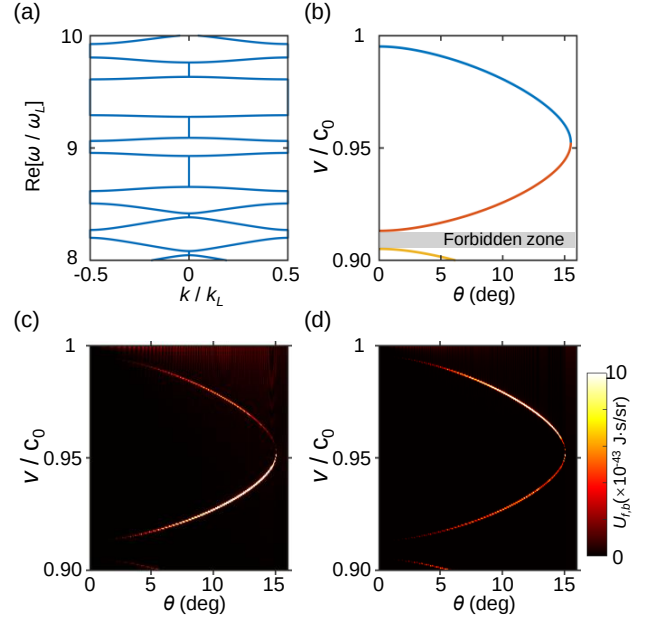


Fig. 2. Angular energy density spectra of the resonance transition radiation generated by the free electron traveling through the moiré lattices. (a) Band structure of moiré lattices. (b) Iso-frequency contour for the wave vector components k_e, k_p of the radiation, i.e., the relation between $v/c_0 = k_0/k_e$ and $\theta = \sin^{-1} k_p/k_0$. (c) Forward and (d) backward angular energy density spectra, respectively.

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