

Polarization Conversion of EM Waves in Time-Varing Magnetized Plasma Medium

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Abruptly time-varying materials enable unprecedented control over electromagnetic (EM) wave properties. This work develops a full-wave finite-difference time-domain (FDTD) framework to analyze polarization conversion in anisotropic, dispersive magnetized plasma with abrupt temporal variations. When the initial EM waves propagate parallel to the ambient magnetic field, linear polarization splits into circularly polarized waves with distinct frequencies. Simulations demonstrate that polarization states can be dynamically manipulated by adjusting the external magnetic field and plasma frequency.

keywords—EM waves, FDTD, Magnetized plasma, Polarization conversion.

I. INTRODUCTION

POLARIZATION represents a fundamental physical property of electromagnetic (EM) waves. The capacity to freely manipulate polarization states is highly desirable, with applications spanning across various fields, including optics, and wave dynamics [1-2]. In contrast to conventional anisotropic homogeneous materials utilized in spatial domains [3-4], materials undergoing abrupt temporal transitions offer a novel degree of freedom for advancing wave engineering, such as frequency and polarization conversion, as well as nonlinear light-matter interaction and so on [5-7].

This study investigates polarization dynamics in magnetized plasma undergoing abrupt temporal transitions. A semi-implicit FDTD method is employed to model wave interactions with time-varying plasma parameters, which provides insights into polarization conversion mechanisms.

II. THEORY AND METHOD

A. Waves Propagation in temporal magnetized plasma

Consider a cold plasma with spatiotemporal inhomogeneity under a static magnetic field $\mathbf{B}_0$ along the direction of wave propagation. Maxwell's equation and constitutive relations govern wave propagation:

$$\nabla \times \mathbf{E} = -\mu_0 \dot{\mathbf{H}} \quad (1)$$

$$\nabla \times \mathbf{H} = \varepsilon_0 \dot{\mathbf{E}} + \mathbf{J} \quad (2)$$

$$\dot{\mathbf{J}} = \varepsilon_0 \omega_p^2(t) \dot{\mathbf{E}} + \omega_b(t) \times \mathbf{J} - \nu_c \mathbf{J} \quad (3)$$

where the ε_0, μ_0 is denote the permittivity and permeability of the free space, respectively. $\dot{\mathbf{A}} = \partial \mathbf{A} / \partial t$. $\omega_p, \omega_b = e|\mathbf{B}_0|/m$, and ν_c denote plasma, cyclotron, and collision frequency, respectively.

B. Semi-Implicit FDTD Equation

This time-varying magnetized plasma systems characterized by the electric density N_e and cyclotron frequency ω_b undergoes abrupt transitions at specific instants while maintaining spatial invariance. Momentum conservation ensures wave number preservation [7]. We align the ambient magnetic $\mathbf{B}_0$ with wave

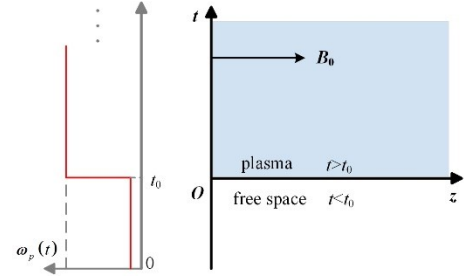


Fig. 1. Schematic of abruptly created temporal plasma.

propagation along $+z$ -axis and adopt a semi-implicit FDTD scheme: explicit iteration for Maxwell's equations (1)-(2) and exponential integrator for the current density equation (3), as formulated in (4)-(7).

$$H_x|_{z+1/2}^{n+1/2} = H_x|_{z+1/2}^{n-1/2} + \frac{\Delta t}{\mu_0 \Delta z} (E_y|_{z+1}^n - E_y|_z^n) \quad (4)$$

$$H_y|_{z+1/2}^{n+1/2} = H_y|_{z+1/2}^{n-1/2} - \frac{\Delta t}{\mu_0 \Delta z} (E_x|_{z+1}^n - E_x|_{z-1}^n)$$

$$E_x|_z^{n+1} = E_x|_z^{n-1} - \frac{\Delta t}{\varepsilon_0 \Delta z} (H_y|_{z+1/2}^{n+1/2} - H_y|_{z-1/2}^{n+1/2}) - \frac{J_x|_z^{n+1}}{\varepsilon_0} \quad (5)$$

$$E_y|_z^{n+1} = E_y|_z^{n-1} + \frac{\Delta t}{\varepsilon_0 \Delta z} (H_x|_{z+1/2}^{n+1/2} - H_x|_{z-1/2}^{n+1/2}) - \frac{J_y|_z^{n+1}}{\varepsilon_0}$$

$$\begin{bmatrix} J_x|_z^{n+1/2} \\ J_y|_z^{n+1/2} \end{bmatrix} = e^{-\nu_c t} \begin{bmatrix} \cos(\omega_{bz} t) & -\sin(\omega_{bz} t) \\ \sin(\omega_{bz} t) & \cos(\omega_{bz} t) \end{bmatrix} \begin{bmatrix} J_x|_z^{n-1/2} \\ J_y|_z^{n-1/2} \end{bmatrix} + \varepsilon_0 \omega_p^2(t) \frac{e^{-\nu_c t}}{\omega_{bz}^2 + \nu_c^2} \begin{bmatrix} C_1 & -C_2 \omega_{bz} \\ C_2 \omega_{bz} & C_1 \end{bmatrix} \begin{bmatrix} E_x|_z^n \\ E_y|_z^n \end{bmatrix} \quad (6)$$

where the C_1 and C_2 denotes as follows

$$\begin{aligned} C_1 &= \nu_c [e^{\nu_c t} - \cos(\omega_{bz} t)] + \omega_{bz} \sin(\omega_{bz} t) \\ \omega_{bz} C_2 &= \omega_{bz} [e^{\nu_c t} - \cos(\omega_{bz} t)] - \nu_c \sin(\omega_{bz} t) \end{aligned} \quad (7)$$

When a traveling wave of frequency ω_0 propagating in free-space encounters a temporal boundary (e.g., permittivity ε_r changes), momentum conservation governs frequency shifts via $n_0^2 \omega_0^2 = n_r^2 \omega_r^2$, where n_0 is the vacuum refractive index, and ω_r and n_r denote the newly created frequency and refractive index

of the time-modulated medium. Abrupt plasma transitions generate six polarization-dependent frequency components [6]: LHCP (O-mode) yields forward waves with frequency of ω_{L1} , and backward waves ω_{L2} and ω_{L3} . The RHCP-incident wave produces forward ω_{R1} and ω_{R2} and backward ω_{R3} , where $\omega_{L1} = -\omega_{R3}$, $\omega_{R2} = -\omega_{L2}$, $\omega_{R1} = -\omega_{L3}$ under collisionless conditions. Critically, FDTD stability necessitates satisfying the Courant-Friedrichs-Lewy condition (CFL) for both initial and generated spectral components.

III. RESULTS AND ANALYSIS

A. Temporal Boundary Interaction

First, FDTD simulations investigate the temporal evolution of waves interacting with a transient magnetized plasma boundary at $t=0$. A circularly polarized plane wave with frequency $\omega_0=2\pi\times 1$ GHz propagates in free-space until the entire space abruptly transitions to magnetized plasma at $t=t_0$, as shown in Fig. 1. Where the plasma frequency $\omega_p=0.8\omega_0$, the cyclotron frequency $\omega_c=0.535\omega_0$, and the collision frequency $\nu_c=10^{-5}\omega_0$. The grid of Exp-FDTD is set as $\Delta z=\lambda_0/80$ and $\Delta t=\Delta z/2c$.

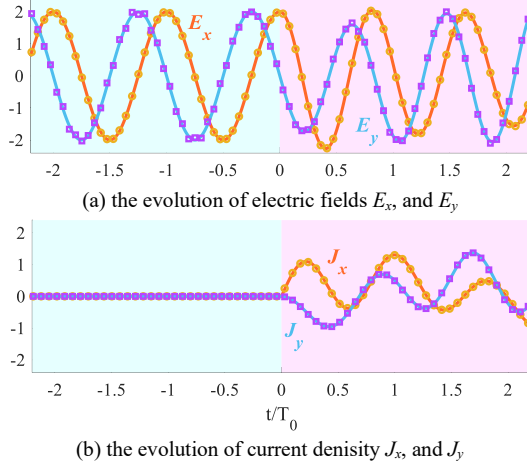


Fig. 2. Time evolution of the E and J when EM waves encounter a temporal magnetized plasma boundary, where the solid curve represents the results in [6] and makers denote the Exp-FDTD results.

As a verification case, Fig. 2 demonstrates that the time evolution of both electric field and current density at the temporal magnetized plasma boundary, computed using the Exp-FDTD method (makers), shows excellent agreement with the Time-Transfer-Matrix Method from [6].

B. Polarization Conversion via Parameter Modulation

In the second part, polarization manipulation through abrupt plasma parameter modulation is examined. The plasma switches on at $t=0$ and off at $t=T_d$, exposing EM waves to two temporal boundaries (electron density rise/fall). For a linearly polarized incident wave with electric field component E_x and frequency $f_0=1$ GHz propagating along the z -axis. Where the plasma frequency, cyclotron frequency and collision frequency of the time-varying magnetized plasma are set as $\omega_p = 3.68\omega_0$, $\omega_c = 8.48\omega_0$, and $\nu_c = 0.05\omega_0$, with duration $T_d = 6.06T_0$.

Fig. 3 demonstrates that the switched magnetized plasma serves as an efficient polarization converter, transforming the

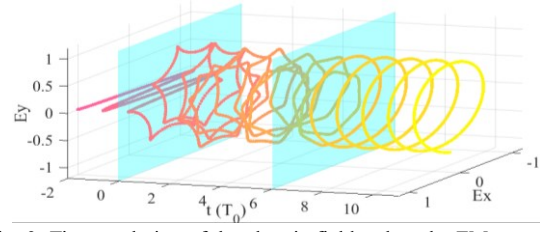


Fig. 3. Time evolution of the electric fields when the EM waves propagation through a temporal magnetized plasma medium, where $\omega_p=3.68\omega_0$, $\omega_c=8.48\omega_0$, and $\nu_c=0.05\omega_0$, the duration of magnetized plasma medium $T_d=6.06T_0$.

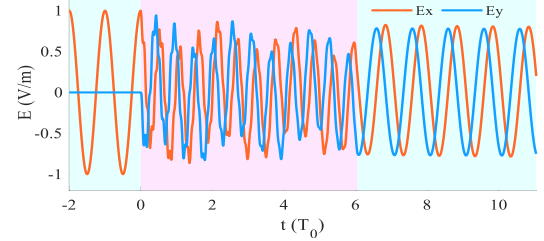


Fig. 4. Time-evolution of electric fields compounds E_x and E_y .

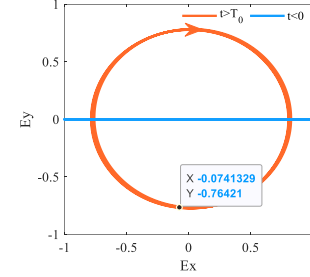


Fig. 5 Polarization properties of the initial wave and output wave.

incident linear polarization ($t < 0$) into LHCP ($t > T_d$). This phenomenon is further elucidated in Fig. 4, which reveals nearly identical amplitudes and a $\pi/2$ phase shift between the E_y and E_x components of the output wave—a hallmark of ideal circular polarization. Additionally, a direct comparison of polarization states between the initial and transmitted waves in Fig. 5 confirms the successful generation of a perfectly achieved LHCP wave mode.

IV. CONCLUSION

This work establishes an FDTD framework to analyze polarization conversion in time-varying magnetized plasma. Abrupt temporal modulation splits linear polarization into LHCP components, with conversion efficiency controlled via plasma frequency and magnetic field adjustments. The methodology provides a foundation for designing dynamic polarization control devices.

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