

Analysis and design of periodic multi-element mmWave metasurfaces by a fast extended method-of-lines formulation

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We introduce a method of lines (MoL) formulation for analyzing and designing periodic multi-element millimeter-wave (mmWave) metasurfaces (MS) in a computationally tractable manner. As the complexity of MSs increases, analysis by full-wave methods becomes challenging. The MoL offers a considerably faster alternative, as it is a semi-analytical method where the electromagnetic equations are solved analytically along the direction perpendicular to the MS layers and numerically on the MS plane, hence decreasing the required degrees of freedom (DoFs). We employ an extended MoL formulation, where a first-order differential equation involving both the total tangential electric and magnetic field components is solved and the transfer matrix of each layer is obtained. As a proof of concept, we analyze various MS filter supercells; a very good agreement with finite element method (FEM) simulations is observed. The presented analysis paves the way for the fast synthesis of optimized MSs by changing only the boundary conditions on the layer interfaces. The advantages of our method will be even more pronounced when designing more complex devices.

Index Terms—computational electromagnetics, electromagnetic metamaterials, metasurfaces, millimeter wave devices, periodic structures

I. INTRODUCTION

METASURFACES are artificial materials that can control propagating waves and can function as absorbers [1], gratings and antennas [2]. Advanced functionalities can be achieved by multilayer [1], [2] or periodic multielement MSs, which provide increased DoFs. Full-wave methods, such as the FEM, provide highly accurate results. However, as the complexity and size of the MSs increase, full-wave methods become computationally challenging. The MoL [3] presents a favorable alternative; it is a semi-analytical method, where the wave equations are solved analytically along the direction perpendicular to the MS and numerically on the MS plane, alleviating the need to discretize the entire 3D domain. The decoupled wave equations are commonly solved for the transverse electric or magnetic field components, with the field components at each layer expressed in terms of the forward and backward propagating waves. This approach has been used in curvilinear devices [5] and multilayer MSs [4], but non-uniform periodic MSs have not been considered. A more general extended formulation [6] has also been introduced, applicable to any medium, even bianisotropic, where the electric and magnetic fields cannot be decoupled. The extended MoL solves a first-order differential equation that involves both the electric and magnetic tangential field; the transfer matrix of each layer is obtained. The extended MoL applications are rather limited, mostly to transmission lines and antennas [6].

In this paper, we extend the MoL formulation, based on a first-order differential equation, to analyze and design periodic multi-element mmWave MSs, comprised of more than one resonator type. Such periodic non-uniform MSs can achieve increased functionalities compared to uniform MSs, hence a fast method for their synthesis is required. As proof of concept, we analyze periodic multi-element MS filters, with a very good agreement to FEM simulations observed in all cases. The MS design can be significantly expedited by the extended MoL

solver, as (i) the computational domain is reduced by one dimension, and (ii) the transfer matrices are calculated only once for each frequency, with different MS patterns introduced on each interface when forming the transfer matrix of the entire device. The advantages of the extended MoL will become more pronounced in the design of even more complex MSs.

II. EXTENDED MoL FORMULATION

The MoL formulation employs Maxwell's normalized equations expressed in terms of the tangential electric and magnetic field components. The coordinates x, y, z are replaced by $x' = k_0 x, y' = k_0 y, z' = k_0 z$, and the magnetic field component intensities $[\tilde{H}_x, \tilde{H}_y, \tilde{H}_z]$ by $\tilde{H}_x = Z_0 H_x, \tilde{H}_y = Z_0 H_y, \tilde{H}_z = Z_0 H_z$. By substituting the longitudinal components $E_z, \tilde{H}_z$ with the transverse ones, we derive the following equations:

$$\begin{aligned} \frac{\partial E_x}{\partial z'} &= -j\tilde{H}_y - \frac{j}{\varepsilon_r} \frac{\partial^2 \tilde{H}_y}{\partial x'^2} + \frac{j}{\varepsilon_r} \frac{\partial^2 \tilde{H}_x}{\partial x' \partial y'}, \\ \frac{\partial E_y}{\partial z'} &= +j\tilde{H}_x + \frac{j}{\varepsilon_r} \frac{\partial^2 \tilde{H}_x}{\partial y'^2} - \frac{j}{\varepsilon_r} \frac{\partial^2 \tilde{H}_y}{\partial y' \partial x'}, \\ \frac{\partial \tilde{H}_x}{\partial z'} &= +j\varepsilon_r E_y + j \frac{\partial^2 E_y}{\partial x'^2} - j \frac{\partial^2 E_x}{\partial x' \partial y'}, \\ \frac{\partial \tilde{H}_y}{\partial z'} &= -j\varepsilon_r E_x - j \frac{\partial^2 E_x}{\partial y'^2} + j \frac{\partial^2 E_y}{\partial y' \partial x'}. \end{aligned} \quad (1)$$

A 2-D discretization scheme based on the Yee cell [7] is considered on the $x - y$ plane. The grid size is $N_x \times N_y$, with N_x, N_y the number of cells along the x and y axes. Periodic boundary conditions (PBCs) are imposed along the x and y axes. They are introduced into the system equations, as they hold for any z . The discretized version of (1) is:

$$\frac{\partial}{\partial z} \begin{bmatrix} \mathbf{E}_x \\ \mathbf{E}_y \\ \tilde{\mathbf{H}}_x \\ \tilde{\mathbf{H}}_y \end{bmatrix} = j \begin{bmatrix} \mathbf{0} & \mathbf{Z} \\ \mathbf{Y} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{E}_x \\ \mathbf{E}_y \\ \tilde{\mathbf{H}}_x \\ \tilde{\mathbf{H}}_y \end{bmatrix} = j\mathbf{A}_s \begin{bmatrix} \mathbf{E}_x \\ \mathbf{E}_y \\ \tilde{\mathbf{H}}_x \\ \tilde{\mathbf{H}}_y \end{bmatrix}. \quad (2)$$

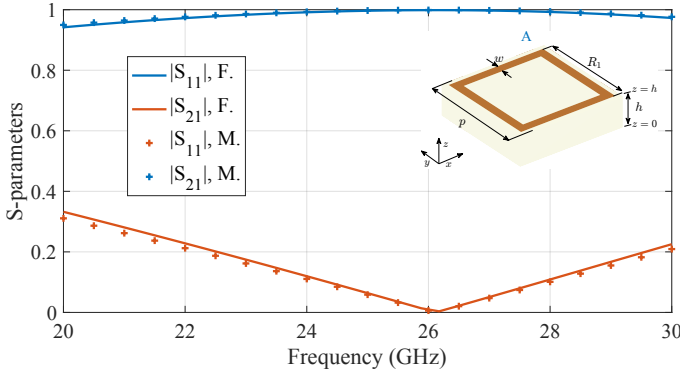


Fig. 1. S-parameter comparison, between the extended MoL (M.) and FEM (F.) for case A (configuration shown as an inset).

The solution to the above equation is a matrix exponential:

$$\begin{bmatrix} \mathbf{E}_x(z) \\ \mathbf{E}_y(z) \\ \tilde{\mathbf{H}}_x(z) \\ \tilde{\mathbf{H}}_y(z) \end{bmatrix} = e^{\mathbf{A}_s k_0 z} \begin{bmatrix} \mathbf{E}_{x0} \\ \mathbf{E}_{y0} \\ \tilde{\mathbf{H}}_{x0} \\ \tilde{\mathbf{H}}_{y0} \end{bmatrix}. \quad (3)$$

The term $e^{\mathbf{A}_s k_0 z}$ is the ABCD matrix and $\mathbf{E}_{x0}$, $\mathbf{E}_{y0}$, $\tilde{\mathbf{H}}_{x0}$, $\tilde{\mathbf{H}}_{y0}$ are the tangential field components on the $z=0$ plane. By multiplying the ABCD matrices of consecutive layers and by applying boundary conditions (BC) on the interfaces, we obtain the transfer matrix of the entire device. Continuity BCs are applied to the tangential electric and magnetic field components, $\mathbf{E}_t/\mathbf{H}_t^{z+} = \mathbf{E}_t/\mathbf{H}_t^{z-}$. Conductive inks of a surface resistance R_s have been recently used as metal surfaces, especially in flexible and conformal MSs. Such metal surfaces can be introduced as a surface current density boundary condition (SCDBC).

$$\tilde{\mathbf{H}}_y^{z+} = \tilde{\mathbf{H}}_y^{z-} - \frac{1}{R_s} Z_0 \mathbf{E}_x^z, \quad \tilde{\mathbf{H}}_x^{z+} = \tilde{\mathbf{H}}_x^{z-} + \frac{1}{R_s} Z_0 \mathbf{E}_y^z, \quad (4)$$

Eq. (4) is applied to the metal cells of each interface instead of the continuity BC. The S-parameters of the MS are calculated as in the decoupled MoL formulation [8].

III. ANALYSIS AND DESIGN OF MS FILTERS

We apply the extended MoL formulation to ring resonator MS unit cells or supercells. We first consider a simpler case (A) of a single unit cell (Fig. 1 inset) of period $p = 2.88$ mm, with a ring resonator of size $R_1 = 2.64$ mm, width $w = 0.24$ mm, $R_s = 0.01 \Omega/\text{sq}$ placed on the top of the substrate ($\epsilon_r = 2.2 - j0.0022$, $h = 0.8001$ mm). We also investigate 2×1 non-uniform MS supercells of size $2p \times p$, termed case B. The MS B consists of two types of ring resonators of size R_1 and R_2 placed on the top of the substrate at $x = p/2$ and $x = 3p/2$ (Fig. 2 inset). We vary the size of the second ring resonator and consider two values $R_2 = 2.16$ mm and $R_2 = 1.92$ mm. A comparison of the S parameters obtained by the extended MoL and FEM is performed for all MSs in Fig.1 (A) and Fig.2 (B), and a very good agreement is observed. The DoFs used for MSs (A) and (B) are 576 and 1152 for the MoL, and approximately 550000 and 960000 for the FEM. As the two MSs of type B are arranged on the same 2×1 substrate, we calculate the ABCD

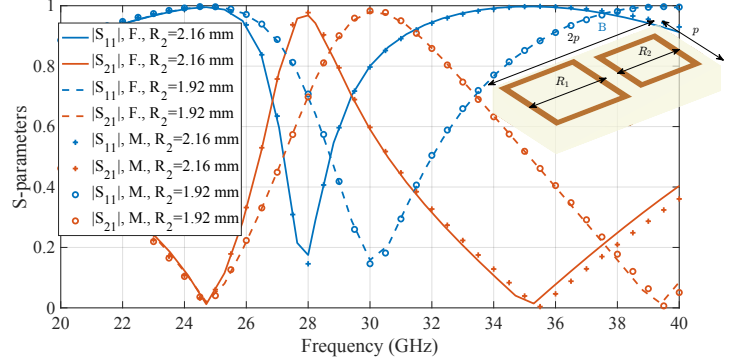


Fig. 2. S-parameter comparison, between the extended MoL (M.) and FEM (F.) for case B (configuration shown as an inset) for two values of R_2 .

matrices of each frequency only once for the two MSs. We then apply the SCDBC on the $z = h$ interface and form the ABCD matrix of each device. Therefore, the design of MSs can be significantly expedited compared to using the FEM; provided that the transfer matrices have been calculated once, the MS response of each different metal pattern can be calculated only by changing the BCs on the $z = h$ interface. In contrast, the FEM would require that we calculate the response of each different MS configuration independently.

IV. CONCLUSIONS

The extended MoL is presented as a means to analyze and design MSs in an efficient and computationally-tractable manner. Our work paves the way for the fast synthesis of larger-scale multi-element MSs with desired functionality.

V. ACKNOWLEDGEMENTS

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