

Conformal Finite-Element Domain-Decomposition Technique for Modeling Wave Propagation in Random Media

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This study introduces a method for modeling large-scale electromagnetic wave propagation by combining the finite element method with a domain decomposition strategy. The Finite Element Tearing and Interconnecting (FETI) technique is used to create consistent system matrices across subdomains with conforming interfaces. This consistency makes it possible to develop surrogate models using regression analysis, allowing for faster computations in complex vegetation scenarios such as forest modeling. The work explores the challenges of building statistical finite element models and validate the idea by solving a synthetic example.

Index Terms—Domain decomposition, finite element analysis, electromagnetic wave-propagation, random media

I. INTRODUCTION

Computing large-scale electromagnetic scattering problems is a significant challenge due to the high computational costs involved. These difficulties are particularly pronounced in applications like vegetation modeling, where the domain must represent random structures such as forests. Generating such randomized models typically requires extensive sampling across large domains, leading to rapidly escalating computational demands [1].

A common method to address this complexity is the use of domain decomposition (DD) techniques [2]. These techniques divide the large problem into smaller, more manageable sub-domains. By solving these individual sub-problems, a statistical sample space can, in principle, be constructed by recombining the results. However, this approach still requires the computation of many sub-models, each capturing different characteristics such as tree shapes, heights, or material properties, which can remain computationally intensive.

An alternative solution is to adopt a DD framework where a single sub-domain acts as a statistical model. This framework captures all necessary variations—such as tree positions, heights, and material parameters—within a single surrogate sub-domain. By eliminating the need to combine multiple sub-models into a large-scale statistical representation, this approach significantly reduces both computational time and resource requirements [3].

While the Finite Element Tearing and Interconnecting (FETI) DD approach has been widely studied [4], [5], its conforming variant has not yet been published, to the authors' knowledge. In certain applications, it is possible to design mesh interfaces between sub-domains to be conforming. This adjustment simplifies the algorithm and has the potential to improve computational performance, making it a promising approach for further exploration, as it is demonstrated in [6] for a DD preconditioner.

In this paper, we introduce a DD framework based on the conforming FETI approach for large-scale electromagnetic

scattering problems, where the domains are arranged in a regular grid pattern. We describe the implementation of the method, examine its computational advantages, and evaluate its performance in scenarios with conforming mesh interfaces. Throughout the study, we ensure that the method is designed to remain compatible with future statistical sub-domain modeling.

II. DOMAIN DECOMPOSITION

Let us consider the following governing equations:

$$\begin{cases} \nabla \times (\frac{1}{\mu_{ri}} \nabla \times \vec{E}_i) - k_0^2 \epsilon_{ri} \vec{E}_i = 0 & \text{in } \Omega_i, \\ \hat{n} \times \nabla \times \vec{E}_i - jk_0 \gamma_t \vec{E}_i = \vec{F}_i^{(inc)} & \text{on } \Gamma_{Ai}, \\ \gamma_t \vec{E}_i = 0 & \text{on } \Gamma_{Gi}, \\ \hat{n} \times \nabla \times \vec{E}_i - jk_0 \gamma_t \vec{E}_i = \vec{\Lambda}_j & \text{on } \Gamma_{ij}, \end{cases} \quad (1)$$

where $\gamma_t \vec{E} = \hat{n} \times (\vec{E} \times \hat{n})$. The index i is specific for a sub-doman, and index j refers to a neighbouring sub-domain. In this model, a perfect conductor ground and scatterer is assumed, represented by Γ_G . Furthermore, Absorbing Boundary Condition (ABC) is assumed on boundaries Γ_A . The excitation is described by $\vec{F}^{(inc)}$, and the coupling between the neighboring surfaces (between sub-domains) is set by $\vec{\Lambda}$.

For the sake of brevity, let us start by considering a finite element method (FEM) equation system for a domain

$$Ax = b, \quad (2)$$

where A is the system matrix, b is the right-hand side (RHS) vector, and x is the vector of unknown degrees of freedom (DoFs). The GetDP FEM framework is used in the recent work [7]. Furthermore, let us assume multiple system matrices representing separate sub-domains. Each sub-domain will have elements in common with others, defining connectivity matrices. To solve the full-domain problem (here illustrated for a two-subdomain example), the following system must be solved:

$$\begin{bmatrix} A_1 & C_{12} \\ C_{21} & A_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}. \quad (3)$$

Following the FETI approach, the number of unknowns is restricted to the sub-domain interface boundaries [8]. This can be expressed as

$$R_i^T R_i = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \quad Z_i = R_i A_i^{-1} R_i^T, \quad (4)$$

where R is the restriction matrix defined by $v = Rx$ [8].

Substituting this into the global problem, we obtain

$$P_{GS}^{-1} Z v = \bar{b}, \quad \text{where } \bar{b} = R \tilde{b}, \quad (5)$$

and P_{GS}^{-1} is the Gauss–Seidel preconditioner, while $\tilde{b}$ is the preconditioned RHS vector. The system in (5) is then solved using a fixed-point iterative solver. This approach effectively enables one to separate the computationally expensive steps (such as the inversion of the system matrices) from the global solution process [8].

III. APPLICATION EXAMPLE

This study demonstrates the proposed method using a straightforward application example in Figure 1, focusing on finite-length perfect electric conductor (PEC) cylinders placed within a cuboid domain surrounded by absorbing boundary conditions (ABC). The PEC cylinder touches the PEC ground. Such simple configurations provide an ideal scenario for quick validation, allowing results to be compared against trusted commercial solvers like HFSS [9]. The example ensures that the proposed conforming DD framework is accurate and reliable. By employing elementary geometries, the method’s computational performance and conformity can be thoroughly validated without introducing unnecessary complexity.

Although this example is intentionally simple to facilitate validation, it serves as a precursor to more complex scenarios that are discussed later in the study and will be detailed in the full paper.

IV. MODEL CONFORMITY

In the conforming DD method, the surface mesh structure on the connecting boundaries remains fixed. As a result, the respective surface elements between two neighboring domains can be paired unambiguously. The impedance matrix Z will only have elements related to the connecting boundary surface elements. Thus, the Z matrices of the same conforming domain, when subject to internal geometric deviations, can be compared element wise between samples (see Figure 1). This feature is not valid for the system matrix A , as internal geometric deviations influence the internal mesh structure.

Consider the arrangement in Figure 1. The cylinder diameter is $1/6$ of the free-space wavelength. In this example, to check the model conformity we introduce a small change in the geometry, we slightly shift the cylinder along a single axis by $\pm 1/120$ of its diameter. To check model conformity, the solution vector of the FETI algorithm is compared resulting in a normalized RMS error is in the range of 1×10^{-3} , indicating

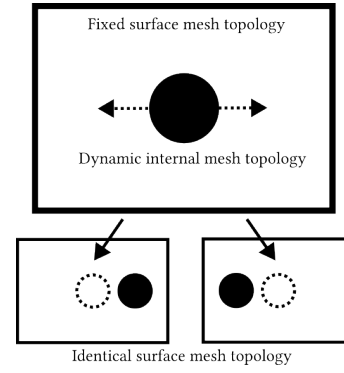


Fig. 1. Concept of the conforming FETI approach, using fixed surface mesh

negligible changes due to the small geometric perturbation. For validation of the results, we compare the real part of the electric field magnitude along one edge of the cuboid. For comparison with a commercial solver (HFSS), the normalized RMS error is 6.2×10^{-2} , showing a good agreement.

V. CONCLUSION

We have presented a statistical DD method tailored for large-scale wave propagation problems in vegetation. The key innovation lies in the conforming FETI approach, which simplifies algorithms and ensures that sub-domain descriptor matrices remain conforming. This property allows these matrices to be leveraged for constructing statistical models, enabling faster calculations. Furthermore, we introduced the framework for statistical model building, paving the way for future applications. Additional data and detailed results will be included in the full paper.

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