

Nonlinear 3D-Magnetostatic Problems Solved with Adjoint-Based Topological Optimization and Volume Integral Formulation

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This paper presents a resolution method to optimize a design problem in 3D-magnetostatic with nonlinear behavior. This method is based on an adjoint approach which is developed in 3D and uses volume integral equations. First, a method for computations of gradients using the adjoint method is presented. Then, in order to validate this method, the adjoint approach is used in an optimization process to design a simple 3D-magnetic circuit.

Index Terms—Integral Equations, Method of Moments, Nonlinear Systems, Optimization Methods, Sensitivity Analysis.

I. INTRODUCTION

Topological optimization is a powerful tool in magnetostatics, widely used to design efficient magnetic structures, [1]. A key challenge in this field is the computational cost associated with gradient-based optimization methods, particularly for complex 3D problems. Adjoint methods have emerged as an effective solution to this challenge by providing an efficient gradient computation framework, [2]. However, their application has often been limited to 2D cases or linear materials. Recent efforts have focused on extending adjoint methods to 3D problems, incorporating non-linear material behaviors, often using finite element formulations, [3]. For magnetic devices with large air volumes, volume integral methods offer an attractive alternative, as they only need to mesh the active regions, which considerably reduces computation time, [4]-[5].

This paper builds upon the magnetic moment method [5]-[6] to develop a 3D adjoint method that directly addresses material non-linearity. By considering saturation effects, this approach enhances the realism of the optimization process, making it better suited for practical applications. The method is validated through optimal design results of a simple 3D magnetic circuit, demonstrating so its ability to handle efficiently non-linear magnetic behavior.

II. PROBLEM FORMULATION

The goal is to design a magnetic structure (region Ω_V), taking into account saturation phenomena, with the objective of reproducing a known field $\mathbf{B}_0$ in a predefined target region Ω_T . The optimization problem addressed in this work is defined as follows:

$$(\mathcal{P}) \left\{ \min_{\rho \in \mathcal{A}} F(\rho) = \int_{\Omega_T} \|\mathbf{B}(\mathbf{M}(\rho), P) - \mathbf{B}_0(P)\|^2 d\Omega_{T_P} \right. \quad (1)$$

where $\rho \in \mathcal{A} = \{L^\infty(\Omega_V) \mid \rho(P) \in [0 ; 1], \forall P \in \Omega_V\}$ and $\mathbf{M} \in (L^2(\Omega_V))^3$. And ρ and $\mathbf{M}$ are linked by the law of materials, Eq. (2), and $\mathbf{B}$ and $\mathbf{H}$ are calculated by the integral equations.

$$\mathbf{M}(P) = \chi_m(\|\mathbf{M}\|, \rho) \mathbf{H}(P, \mathbf{M}) \quad P \in \Omega_V \quad (2)$$

In this formulation, ρ represents the material density. A value of $\rho(P)$ close to 0 indicates that P is air, while a value close to 1 corresponds to the ferromagnetic material. The constraint

enforces the material law, which links the magnetization $\mathbf{M}$ to the magnetic field $\mathbf{H}$ through the magnetic susceptibility χ_m . Saturation phenomena influence the variation of χ_m , which is why χ_m depends on the magnetization $\mathbf{M}$, allowing the non-linear behavior of the material to be taken into account.

A. Sensitivity analysis with Adjoint Method for a nonlinear Problem

The gradient of the function associated to the problem $(\mathcal{P})$ is:

$$\forall \rho \in [0 ; 1], \quad D_\rho F(\rho) = -\mu_0^2 \int_{\Omega_V} \boldsymbol{\lambda}(P) \frac{\partial \chi_m}{\partial \rho} \mathbf{H}(d, P) d\Omega_V \quad (3)$$

where $\boldsymbol{\lambda} \in (L^2(\Omega_V))^3$ is the solution of the adjoint problem :

$$\begin{aligned} \boldsymbol{\lambda}(P) - \frac{\partial \chi_m(\|\mathbf{M}\|, \rho)}{\partial \|\mathbf{M}(P)\|} \frac{\mathbf{H}(\mathbf{M}, P) \cdot \boldsymbol{\lambda}(P)}{\|\mathbf{M}(P)\|} \mathbf{M} - \mathbf{L}_{\Omega_V}(\chi_m(\|\mathbf{M}\|, \rho) \boldsymbol{\lambda}, P) \\ = -\mathbf{L}_{\Omega_T}(\mathbf{M}_a, P) \quad , \forall P \in \Omega_V \end{aligned} \quad (4)$$

with $\mathbf{M}_a(\mathbf{M}, P) = 2 \frac{\mathbf{B}(\mathbf{M}, P) - \mathbf{B}_0(P)}{\mu_0}$ and the operator $\mathbf{L}_X$ as follows:

$$\begin{aligned} \mathbf{L}_X : (L^2(X))^3 \times \Omega \setminus X &\longrightarrow \mathbb{R}^3 \\ (\mathbf{M}, P) &\longmapsto \\ \frac{1}{4\pi} \int_X \left(\frac{3\mathbf{M}(Q) \cdot (\mathbf{r}_P - \mathbf{r}_Q)}{\|\mathbf{r}_P - \mathbf{r}_Q\|_{\mathbb{R}^3}^5} (\mathbf{r}_P - \mathbf{r}_Q) - \frac{1}{\|\mathbf{r}_P - \mathbf{r}_Q\|_{\mathbb{R}^3}^3} \mathbf{M}(Q) \right) dX_Q \end{aligned} \quad (5)$$

Compared to the sensitivity analysis theorem associated with a linear problem (see [6]), nonlinearity appears through the derivative of χ_m with respect to the magnetic moment $\mathbf{M}$. This derivative arises during the differentiation of the Lagrangian function. However, what is particularly interesting about using the adjoint method for solving a nonlinear problem, is that the gradient computation still requires solving only a linear system. Even with the inclusion of the term involving the derivative of χ_m , the structure of the adjoint problem remains unaffected, preserving its linear nature. For gradient computation using this method, the direct calculation of F involves employing a nonlinear resolution method, such as the fixed-point method or the Newton-Raphson one. By using the results from this nonlinear resolution, it becomes possible to solve the associated linear adjoint problem and compute the gradient of F .

B. Discretization Validation of the Gradient Computations

The validation of the gradient calculation by using the adjoint method from the previous theorem is done by comparing some numerical results with the finite difference method. To compute the gradient of function F of problem $(\mathcal{P})$ using the finite difference calculations. All tests were performed on an electromagnet ($H = 15\text{cm}$ and $L = l = 5\text{cm}$) with a coil which surround it. For this structure, a mesh consisting of 2,000 elements was used. Test material density is 0.5 throughout Ω_V . On average, the error per element was 1.6%, with a maximum of 5.5%. The computation time for a calculation using the adjoint method was 101 seconds, compared to 161 494 seconds (44 h) for a finite difference calculation. This means that the adjoint method is approximately 1 598 times faster than the finite difference approach. These values provide an estimate of the time savings; however, it is important to note that these durations also depend on the specific nonlinear resolution method employed and on its accuracy. For this test, the Newton-Raphson method was used. However, the fixed-point method is also implemented and also provides efficient results. However, for devices exhibiting significant saturation, the Newton-Raphson method is preferable due to its rapid convergence in a limited number of iterations (even if each iteration takes longer than the fixed-point method).

III. OPTIMAL 3D-DESIGN RESULT

In this section, we will use the sensitivity analysis theorem to solve an optimization problem of design. For this purpose, the field $\mathbf{B}_0$ was simulated using a specific structure with linear material that should be recovered at the end of the optimization process (this is an inverse problem). The fact that we're using a non-linear resolution will show whether there's a difference between linear and non-linear. The SIMP method (Solid Isotropic Material with Penalization) with a polynomial penalization $\chi_m(\|\mathbf{M}\|, \rho) = \chi_m(\|\mathbf{M}\|)\rho^d$ were chosen to ensure a physically feasible final result, [1]-[3]-[6].

A. Results

The field $\mathbf{B}_0$ is now computed using a reference structure composed of a linear magnetic material. We chose this configuration because the thin parts of the variable design zone are likely to cause saturation if a ferromagnetic material were used. As a result, we expect the optimized solution to differ significantly. The initial condition is set with $\rho = 0.5$ throughout the variable mesh. The reference structure and the final result after the optimization process are shown in Figure 1.

B. Convergence

Some key points regarding the convergence of the optimization process are presented in Table I.

The optimization starts from a homogeneous state with $\rho = 0.5$ in the variable mesh. A penalization factor increasing from 3 to 7 every 100 iterations is used to progressively enforce a binary distribution and promote realistic designs. At each iteration, the nonlinear magnetostatic problem is solved using Newton-Raphson. If it fails to converge, a fixed-point method is used as a fallback. The solution from the previous iteration

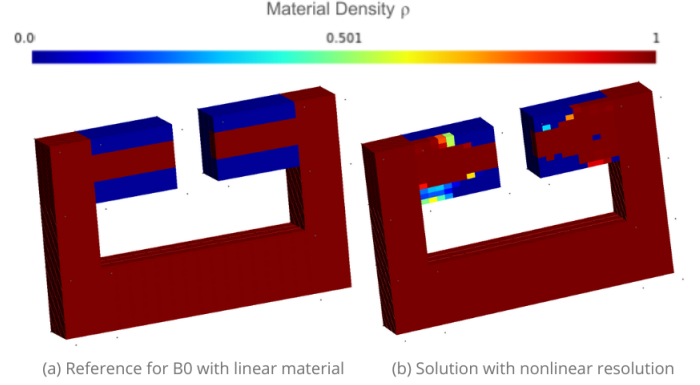


Fig. 1. Expected solutions and optimized results in linear and non linear

TABLE I
CONVERGENCE FOR THE OPTIMIZATION PROBLEM

Resolution	Iterations	F
Starting Point ($\rho = 0.5$)	0	9×10^5
Nonlinear Solution, $d = 3$	150	2515
Nonlinear Solution, $d = 7$	700	2003

serves as the initial guess, ensuring stability and efficiency. As seen in Table I, the nonlinear model leads to a limited decrease in the objective function, which is expected due to saturation effects. Despite a higher objective value, the final structure is more physically accurate, especially in narrow regions where saturation significantly impacts the field.

IV. CONCLUSION

In conclusion, this paper presents an adjoint method based on the Magnetic Moment Method, integrated into the resolution of topology optimization problems in magnetostatic for 3D nonlinear scenarios. The adjoint method was tested and validated by comparing it to finite difference calculations. It was then applied to solve Problem $(\mathcal{P})$ corresponding to the design of a 3D-magnetic circuit, and this method successfully reconstructs the a priori known structure. The structure used in this paper represents a simple 3D-case study, and the method must be adapted for application to more complex devices.

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