

# A Topology Optimization Framework Employing Closed NURBS Curve-Controlled Boundaries and Curved Triangle Remeshing

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This paper proposes a new topology optimization (TO) framework that employs closed Non-uniform rational B-spline curve (CNRBC)-controlled boundaries and curved triangle remeshing (CTR). The proposed CNRBC can represent the closed boundaries of material with nearly any shape and control the boundaries of material with few parameters, and both the stochastic methods and gradient methods can be easily applied to optimize the control parameters. The proposed CTR can accurately describe the curve boundaries of CNRBC based on an initial fixed mesh, and the time-consuming remeshing for the whole domain is avoided. The proposed CNRBC-CTR is applied to the TO of a magnetic actuator to verify its effectiveness.

**Index Terms**—closed NURBS curve, curved triangle, remeshing, topology optimization.

## I. INTRODUCTION

Topology optimization (TO) aims to optimize material without prior knowledge, and has gained great attention [1]. The commonly variable density method needs fine mesh to obtain clear boundaries, which can cause low optimization efficiency [2]. The level set method can obtain clear boundaries, but it is prone to generating jagged structures [3], and more efforts are needed to smooth boundaries. What is more, the above methods need to calculate the derivative information to update, which render difficult implementation.

Other boundary class methods can acquire novel structures, but the fixed topology of basic components makes it difficult to obtain arbitrary smooth material structures [4]. Particularly, the current methods usually adopt fixed mesh in the design domain, and the calculation accuracy under different material structures is not considered.

In this work, a topology optimization framework employing closed Non-uniform rational B-spline (CNRBC) curve (CNRBC)-controlled boundaries and curved triangle remeshing (CTR) is proposed. The smooth and arbitrary boundaries with few control parameters can be obtained with CNRBC, and both the stochastic method and gradient method can be applied to optimize the control parameters. Furthermore, we can accurately describe the curve boundaries of CNRBC with CTR. The TO results of a magnetic actuator verify the effectiveness of our proposed CNRBC-CTR method.

## II. MATERIALS AND METHODS

### A. Equations of NURBS curve

To represent any geometric boundaries, the NURBS curve is introduced first, which can be described as follows [5]:

$$P(u) = \frac{\sum_{i=0}^n b_i \omega_i N_{i,k}(u)}{\sum_{i=0}^n \omega_i N_{i,k}(u)}, \quad (t_{k-1} < u < t_{n+1}) \quad (1)$$

where  $k$  represents the order,  $\omega_i$  represents the weight factor ( $i = 0, 1, \dots, n$ ).  $b_i$  is the control point.  $\Xi = [t_0, t_1, t_2, \dots, t_{n+k}]$  represents the knot vector, which is non-decreasing.  $N_{i,k}(u)$  is the basis function.

Once the  $n+1$  control points, the order  $k$ , and the  $n+1$  weight factors are given, the NURBS curve can be determined. The advantage of NURBS curve is that its  $C^{k-2}$  continuous within the node interval and  $C^{k-s-1}$  continuous at the node, where  $s$  represents the degree of repetition at the node.

### B. Closed NURBS curve

The boundary of the processable entity must be a closed curve. The simplest closed curve is the circle, and the circle can be determined when the radius  $R$  and the coordinates  $(x_0, y_0)$  of the center point  $O$  are given. For any point  $M$  on the circle, if the angle between the line  $OM$  and the  $+X$  axis is  $\theta$ , then the coordinates of point  $M$  can be described as follows:

$$\begin{cases} x = R \cos(\theta) + x_0 \\ y = R \sin(\theta) + y_0 \end{cases}, \quad (\theta \in [-\frac{\pi}{2}, \frac{3}{2}\pi]) \quad (2)$$

Therefore, the shape of the circle can be changed by adjusting the radius  $R$  and the center coordinates. To maintain the smoothness of different points with different radius, the following closed NURBS curve (CNRBC) is introduced:

$$R(\theta) = \frac{\sum_{i=0}^n R_i \omega_i N_{i,k}((\theta + \pi/2)/2\pi)}{\sum_{i=0}^n \omega_i N_{i,k}((\theta + \pi/2)/2\pi)}, \quad (\theta \in [-\frac{\pi}{2}, \frac{3}{2}\pi]) \quad (3)$$

Where the radius of start is equal to that of the end point, that is,  $R_0 = R_n$ . Based on the proposed CNRBC, we can obtain a closed curve with nearly any shape. To maintain the smoothness at the starting point, the radius of the starting point is the average of the radius of two adjacent control points.

Fig. 1 shows the CNRBCs under different control radius vectors, where 9 control radii are adopted. In Fig. 1(a), all radii and weights are 2 and 1, and the CNRBC is a circle. In Fig. 1(b), by changing the radius vector, the peach-shaped

CNRBC is obtained. Therefore, the CNRBC can easily fit more complex closed boundaries by adjusting center coordinates, the radius vector, and the weight vector. Particularly, more complex structures with holes can be easily achieved by implementing CNRBCs and Boolean operations.

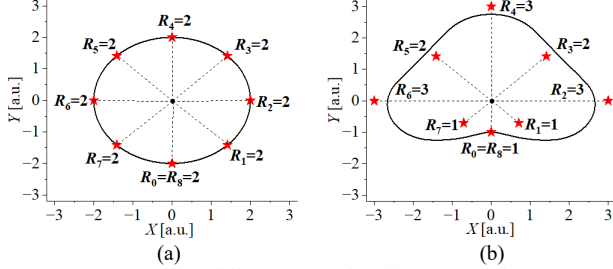


Fig. 1. CNRBCs under different control radii. (a) Circular CNRBC. (b) Peach-shaped CNRBC.

### C. Remeshing techniques

Remeshing for the whole domain according to different CNRBCs is time-consuming. In Fig. 2(a), the initial fixed mesh is adopted in the design domain. If the element is within the CNRBC, then the color is gray (material 1); otherwise, the color is white (material 2). We can see that some elements are passed through by the CNRBC, and the initial FEM meshes cannot accurately describe the curve boundary of the CNRBC, and there can be errors in estimating the performance of the material topology, especially for devices that are sensitive to the structural boundaries.

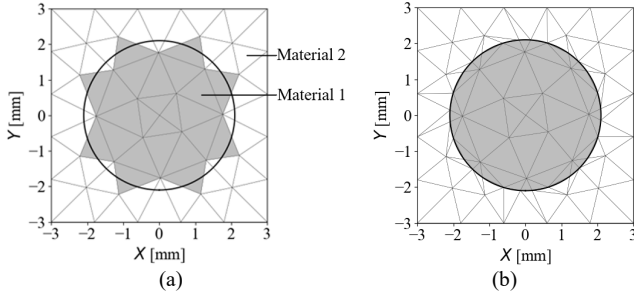


Fig. 2. Mesh in design domain. (a) The Relationship between FEM mesh and CNRBC. (b) New FEM mesh with curve edges and CNRBC.

Fig. 3(a) presents a triangle passed through by the CNRBC (red curve). The proposed CTR technique is presented in Fig. 3(b). The original triangle is divided into 3 second-order triangles, and the isoparametric mapping approach is adopted to obtain the element stiffness matrix. As a result, the boundaries of CNRBC can be accurately described by the divided elements. This is completely different from existing works, where the numerous elements with smaller straight edges are adopted to approximate the curve, resulting in approximation errors and low calculation efficiency. The newly FEM mesh with curved edges and the CNRBC are presented in Fig. 2(b).

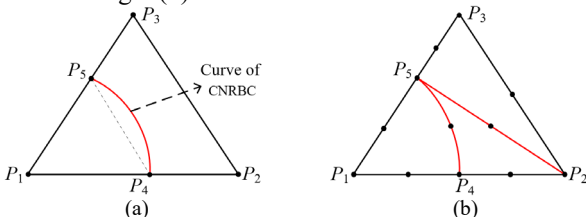


Fig. 3. Curved triangle remeshing. (a) Original triangle. (b) Curved triangles after remeshing.

## III. NUMERICAL EXAMPLE

The proposed CNRBC-CTR method is applied to the design of a magnetic actuator [3], where the topology of the ferromagnetic material (FM) needs to be determined within the design domain (the rectangular area corresponding to the red dashed line). The goal is to maximize the magnetic force of the armature under a volume fraction (VF). The magnetic force is calculated by integrating Maxwell's stress tensor over the exterior surfaces  $S$  of the armature.

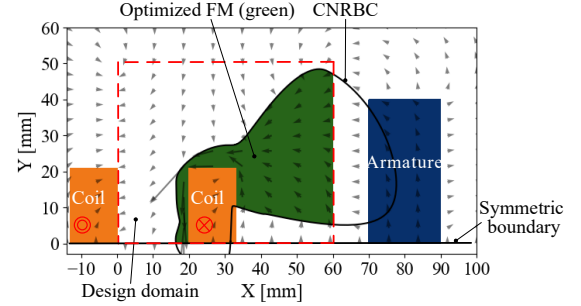


Fig. 4. Symmetrical model of magnetic actuator, and the optimal FM under the VF of 0.4 is the green part. (Unit: mm).

Initially, a single CNRBC with only 10 control radii is adopted. The optimal parameters, including control radii, weights, and center coordinates of CNRBC, are searched utilizing the method of moving asymptotes. Fig. 4 presents the TO results under the VF of 0.4, and the magnetic force corresponding to the optimized FM is 33.50 N/m. We can see that the FM structure with smooth boundaries and high performance is obtained with our proposed method. It is worth noting that only a single CNRBC and a few control radii are utilized here, and the designed FM may not be the optimal. Next, we will explore more control parameters and more CNRBCs, which will be presented in the full paper.

## IV. CONCLUSION

This study proposed a topology optimization framework utilizing CNRBC-controlled boundaries and CTR technique, which can ensure both the calculation accuracy and efficiency. The results of the numerical example show that the proposed CNRBC-CTR method can easily obtain the smooth boundaries of material with high performance.

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