

A New Optimization Methodology Based on Material-Field Series Expansion for Topology Designs of Electromagnetic Devices

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Topology design is the concept design of a product, which can create a novel topology of the product that might be “unthinkable” beforehand. Topology Optimization (TO) is increasingly recognized as the paradigm of the predominant engineering techniques to provide a quantitative design method for modern engineering design. Nevertheless, the extremely high dimensionality and overwhelmingly heavy computational burden are the main factors restricting the implementation of TOs in engineering practice. To address the aforementioned issues, a TO methodology based on material-field series expansion (MFSE) is proposed. Based on the MFSE, the structural topology is represented by a newly-introduced unbounded material field with spatial dependency and the TO problem is approximated as a constrained model with the truncated series expansion coefficients as the design variables, significantly reducing the dimensionality of design variables and inherently avoiding the checkerboard pattern without any additional measure. The numerical example is given to validate the feasibility and effectiveness of the proposed methodology and demonstrate its advantage for dimensionality and computational cost reductions in TO designs.

Index Terms—Finite element method (FEM), material-field series expansion (MFSE), tabu search algorithm (TSA), topology optimization (TO).

I. INTRODUCTION

TOPOLOGY design is the highest level, or concept design of a product, which can create a novel topology of the product that might be “unthinkable” beforehand. Consequently, topology optimization (TO) is increasingly recognized as the paradigm of the predominant engineering techniques to provide a quantitative design method for modern engineering design.

Nevertheless, it is only after the introduction of finite element method (FEM) into TO procedure that TO techniques can be used to solve structure TOs accurately. Moreover, the gradient information is commonly needed when updating the material of an element. All these factors are adding up to present a heavy burden on the computation resources to make TO an overwhelmingly complex computational task. In this point of view, the existing TO techniques, including the level-set method [1], the ON/OFF method [2], and the stochastic algorithm based ones [3]–[4], are all computationally inefficient.

In this regard, a methodology based on material-field series expansion (MFSE) is proposed to significantly reduce the number of degrees of freedom in a TO procedure. Moreover, a promising byproduct of the proposed methodology is its inherent checkerboard pattern free characteristics.

II. A NOVEL METHODOLOGY FOR TOPOLOGY OPTIMIZATION

In a TO procedure using an existing TO technique, the structure is firstly meshed into elements, and the material for each element is then iteratively updated. In other words, the number of the degrees of freedom in a TO design is equal at least to the number of the elements. Considering the fact that the number of the total elements for even a 2D structure will reach at least several thousands, a TO design is generally overwhelmingly computationally expensive. In this regard, a novel methodology for TOs based on MFSE is proposed.

In the proposed methodology, the structural topology can be defined by a material-field distribution assuming that each

element is occupied by either a solid or a void material. Different from existing works, an unbounded material-field function $\varphi(\mathbf{x})$ is introduced to facilitate implementations, i.e.,

$$\begin{cases} \varphi(\mathbf{x}) \leq 0, \text{ void material} \\ \varphi(\mathbf{x}) > 0, \text{ solid material} \end{cases} \quad (1)$$

where $\mathbf{x}$ is the center position of the element.

Different from a traditional TO method, a distance-related function is introduced to represent the spatial dependency of the material field in the design domain and assumed to have an exponential form as [5]

$$C(\mathbf{x}_i, \mathbf{x}_j) = \exp(-\|\mathbf{x}_i - \mathbf{x}_j\|^2 / l_c^2) \quad (2)$$

where $C(\mathbf{x}_i, \mathbf{x}_j)$ represents the spatial dependency between the material-field function values at two positions/elements $\mathbf{x}_i$ and $\mathbf{x}_j$; l_c is the correlation length of the material field, generally set to 0.3 of the minimum length of the design domain.

If the design domain is discretized into N_p elements with N_p observation points in the element centers, a distance-related matrix $\mathbf{C}$ for the N_p centers is formulated as

$$\mathbf{C} = \begin{bmatrix} C(\mathbf{x}_1, \mathbf{x}_1) & \cdots & C(\mathbf{x}_1, \mathbf{x}_{N_p}) \\ \vdots & \ddots & \vdots \\ C(\mathbf{x}_{N_p}, \mathbf{x}_1) & \cdots & C(\mathbf{x}_{N_p}, \mathbf{x}_{N_p}) \end{bmatrix} \quad (3)$$

In the MFSE method, the material-field function $\varphi(\mathbf{x})$ can be represented by using the eigenfunctions of the distance-related matrix as basis functions, namely

$$\varphi(\mathbf{x}) = \sum_{j=1}^{N_p} \eta_j \frac{1}{\sqrt{\lambda_j}} \boldsymbol{\psi}_j^T \mathbf{C}_d(\mathbf{x}) \quad (4)$$

where η_j are the MFSE coefficients to be determined, λ_j and $\boldsymbol{\psi}_j$ are the j th eigenvalue and normalized eigenvector, respectively, of the distance-related matrix $\mathbf{C}$ sorted in descending order; and $\mathbf{C}_d(\mathbf{x}) = [C(\mathbf{x}, \mathbf{x}_1) \cdots C(\mathbf{x}, \mathbf{x}_{N_p})]^T$.

Since the eigenvectors with smaller eigenvalues have less influence on the material-field function, it is computationally efficient to truncate (4) to the M largest principal eigenvalues of the corresponding eigenvectors under a given truncation error ε . Moreover,

$$1 - \sum_{j=1}^M \lambda_j / \sum_{j=1}^{N_p} \lambda_j \leq \varepsilon \quad (5)$$

The truncated series expansion of the material-field function is expressed as

$$\varphi(\mathbf{x}) \approx \sum_{j=1}^M \eta_j \frac{1}{\sqrt{\lambda_j}} \boldsymbol{\psi}_j^T \mathbf{C}_d(\mathbf{x}) = \boldsymbol{\eta}^T \boldsymbol{\Lambda}^{-1/2} \boldsymbol{\Phi}^T \mathbf{C}_d(\mathbf{x}) \quad (6)$$

where $\boldsymbol{\eta} = [\eta_1, \dots, \eta_M]^T$ is the vector of the unconstrained design variables of the TO methodology based on the MFSE, $\boldsymbol{\Lambda} = \text{diag}(\lambda_1, \dots, \lambda_M)$, $\boldsymbol{\Phi} = [\boldsymbol{\psi}_1, \dots, \boldsymbol{\psi}_M]$.

Under a given truncation error ε of 0.01, M is in the region of [50, 100]. Obviously, compared with a traditional TO method, the proposed one significantly reduces the dimensionality of the design variables from N_p to M , and inherently avoids the checkerboard pattern.

III. NUMERICAL EXAMPLE AND CONCLUSIONS

To validate the feasibility and effectiveness of the proposed methodology and compare its performance with existing techniques, the TO of an electromagnetic actuator is selected as a case study [3],[4].

First, to compare performance with existing techniques, the same mesh of the design domain of 270 elements as given in [3],[4] is used. In the implementation of the proposed MFSE based methodology, a truncation error ε of 0.01 reduces the original 270 degrees of freedom to $M = 69$ design variables, and a tabu search algorithm (TSA) is then used to update iteratively these 69 design variables. The electromagnetic force in the specified direction on the armature is computed by FEM and is maximized.

The finally optimized result and the performance of the proposed methodology (MFSE+TSA) are given in Table I. Also, the corresponding results and performances of existing techniques including the traditional ON/OFF method (ON/OFF) [2], the hybrid methodology combining ON/OFF method and genetic algorithm (GA) [3], the methodology based on revised quantum-inspired evolution algorithm (RQEA) [4] are also given in the same table. For a fair comparison, the electromagnetic forces are re-calculated using the optimized topologies of these existing works under the same mesh.

TABLE I
OPTIMIZATION RESULTS AND PERFORMANCES COMPARISON

Optimization method	Electromagnetic force (N/m)	CPU time (s)
ON/OFF	-49.22	9385.783
ON/OFF+GA	-54.36	31870.162
RQEA	-54.39	107843.47
MFSE+TSA	-54.41	8344.287

From these numerical results, it is obvious that: (1) in view of the convergence speed, the proposed methodology is faster than the fastest existing local approach: the ON/OFF method but with an approximately 11% increase in the optimized

electromagnetic force; (2) in view of the global search ability, the proposed methodology finds the so far most maximum force, slightly better than that of the existing stochastic algorithm based method: RQEA-based method but uses only one-thirteenth of the CPU time of the latter, since the dimensionality of design variables is reduced to nearly one-fourth of its original size.

To further demonstrate the advantage of the proposed methodology for dimensionality reduction in a high-dimensional TO, the elements (observation points) of the design domain are expanded from 30×9 to 150×45 (6750). Under the same truncation error of 0.01, the original 6750 design variables reduce to $M = 71$ ones, a merely increasing of 2 from 69. Table II tabulates the optimized results and performances of the proposed methodology in different iterative cycles. Again, the proposed methodology can significantly reduce the computational cost. Fig. 1 presents the convergence trajectories of the optimized topologies in the proposed TO procedure. Obviously, there is no checkerboard pattern in the finally optimized topology of the proposed methodology although no special technique is used.

TABLE II
THE VARIATION OF OPTIMIZATION RESULTS WITH ITERATION

Iteration	Electromagnetic force (N/m)	CPU time (s)
200	-55.72	11916.554
500	-55.95	29352.559
800	-56.07	48476.980

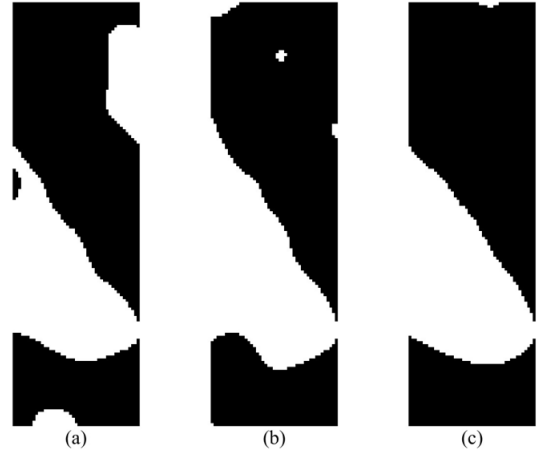


Fig. 1. The convergent trajectories of the optimized topologies in the proposed TO procedure. (a) Iteration = 200. (b) Iteration = 500. (c) Iteration = 800.

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