

Multi-objective Optimization of a Split-tooth Permanent Magnet Vernier Motor Using Response Surface Model and NSGA II

Ruixiang Liu¹, Fei Zhao¹, Ping Wu¹, Xiang Cao¹, Chao Zhi², and Haotian Wu³

¹School of Robotics and Advanced Manufacturing, Harbin Institute of Technology (Shenzhen), Shenzhen, 518055 China

²Meituan Technology, Huilong Business Center, Longhua District, Shenzhen, 518100 China

³Meituan Technology, Taiji Building, Chaoyang District, Beijing, 100102 China

This paper proposes a multi-objective optimization design method for a split-tooth permanent magnet vernier motor (ST-PMVM) to enhance the motor's torque output capacity and short-term overload capacity. The torque output capacity and overload capacity are firstly defined. The key variables are selected by the sensitivity analysis. Subsequently, the response surface model (RSM) is integrated with the Non-dominated Sorting Genetic Algorithm II (NSGA-II) to generate a Pareto-optimal solution set for the sensitive variables, from which the optimal design variables are determined. Finite element analysis (FEA) is employed to compare the electromagnetic performance of the initial and optimal motor designs. The results show that the optimal ST-PMVM demonstrates a 13.6% enhancement in average torque, a 9.6% improvement in torque density, and a 68.8% and 120% extension in maximum operation time under 2 times and 2.5 times overload conditions, respectively, demonstrating the effectiveness of the proposed optimization method.

Index Terms—Split-tooth permanent magnet vernier motor, sensitive analysis, response surface model, non-dominated sorting genetic algorithm II.

I. INTRODUCTION

With the rapid advancement of robotic technology, the demand for high-performance joint motors in robotic arms has become increasingly critical [1]. As a key component of robotic systems, robotic arm joint motors are required to meet stringent design specifications, including high torque density, efficient energy utilization, robust short-term overload capability and so on [2]. Among the various motor topologies, permanent magnet vernier motors (PMVMs) have emerged as a promising solution due to their inherent advantages of high torque density, and simplified mechanical structure [3]. However, the performance requirements for high torque output and overload capacity in joint motors often conflict with one another, presenting significant challenges in motor design.

This paper proposes a multi-objective optimization method for split-tooth permanent magnet vernier motor (ST-PMVM) to address the inherent design conflict between torque output and overload capacity in joint motor applications. The optimal design variables fulfilling the requirements of high torque output and high overload capacity are obtained. The results indicate that the proposed method significantly enhances the motor's torque output and short-term overload capability.

II. MULTI-OBJECTIVE OPTIMIZATION

A. Design Variables and Optimization Objectives

The design variables requiring optimization for the ST-PMVM are shown in Fig. 1. On this basis, a parametric model for the ST-PMVM is established. The optimization objectives of ST-PMVM are defined as the average torque T_{avg} , torque density T_{den} and short-term overload capacity based on practical robotic applications. To evaluate the overload capacity, the overload coefficient k is defined as:

$$k = \frac{T_h}{hT_N} \quad (1)$$

where h is the multiple of overload current, T_N represents the rated torque and T_h represents the torque output under h -times overload current. The overload coefficient k reflects the linearity of the motor's torque-current (T - I) curve. A higher value of k indicates that the T - I curve is closer to an ideal linear state, which means that less current is required to achieve a certain overload torque. For the robotic arm application, operation under 2-3 times overload conditions is required. Hence, h is set to 3 in this study.

B. Comprehensive Sensitivity analysis

The sensitivity indices of each design variable on three design objectives are calculated based on 25 samples generated using definitive screening design method (DSD). A comprehensive sensitivity index $G(n_i)$ is introduced to evaluate the sensitivity of different design variables. The comprehensive sensitivity index $G(n_i)$ can be expressed as:

$$G(n_i) = \omega_1 |S_{T_{avg}}| + \omega_2 |S_{T_{den}}| + \omega_3 |S_k| \quad (2)$$

where $S_{T_{avg}}$, $S_{T_{den}}$, and S_k are the sensitivity of average torque, torque density, and overload coefficient, respectively. Besides, ω_1 , ω_2 , and ω_3 represent the weight coefficient of the average torque, torque density, and overload coefficient, respectively, which satisfy $\omega_1 + \omega_2 + \omega_3 = 1$. ω_1 , ω_2 , and ω_3 are selected to be 0.2, 0.4, and 0.4, respectively, based on performance requirements. The sensitivity indices of 11 design variables for the three design objectives are shown in Table I. The design variables h_{s0} , h_{sy} , h_{PM} and θ_{s0} exhibit high comprehensive sensitivity and should be prioritized for optimization in the multi-objective optimization process.

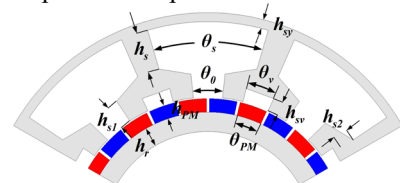


Fig. 1. Geometric parameter model

TABLE I
COMPREHENSIVE SENSITIVITY INDEXES

Design variables		Design objectives			G_{ni}
		$S_{T_{avg}}$	$S_{T_{den}}$	S_k	
x_1	θ_s	0.071	0.083	0.024	0.057
x_2	θ_{sv}	0.052	0.064	0.008	0.039
x_3	θ_{s0}	0.616	0.671	0.290	0.508
x_4	θ_{PM}	0.059	0.074	0.043	0.059
x_5	h_{PM}	0.052	0.074	0.168	0.107
x_6	h_{sy}	0.259	0.224	0.142	0.198
x_7	h_{s0}	0.234	0.260	0.079	0.182
x_8	h_{s1}	0.085	0.088	0.018	0.059
x_9	h_{s2}	0.039	0.042	0.013	0.030
x_{10}	h_{sv}	0.117	0.114	0.041	0.085
x_{11}	h_r	0.059	0.083	0.022	0.054

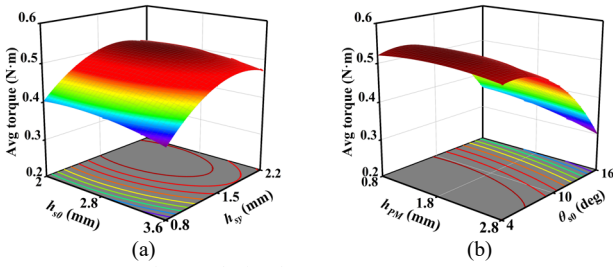


Fig. 2. Response surface analysis of ST-PMVM

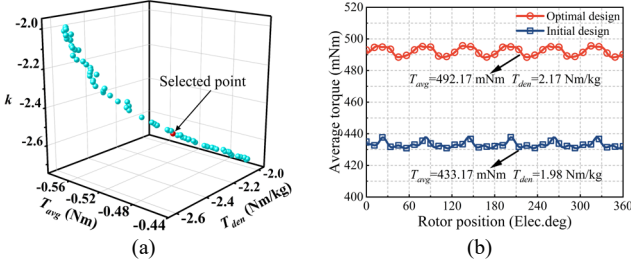


Fig. 3. (a) Pareto solution set of three optimization objectives (b) Initial and optimal average torque

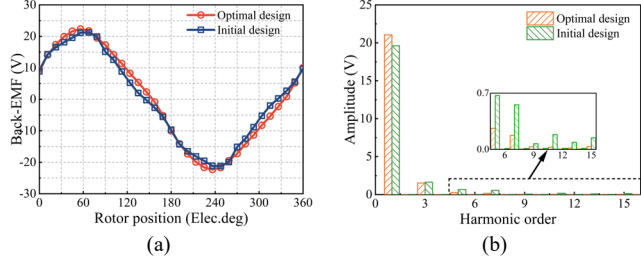


Fig. 4. Initial and optimal back EMF. (a) Waveform. (b) THD

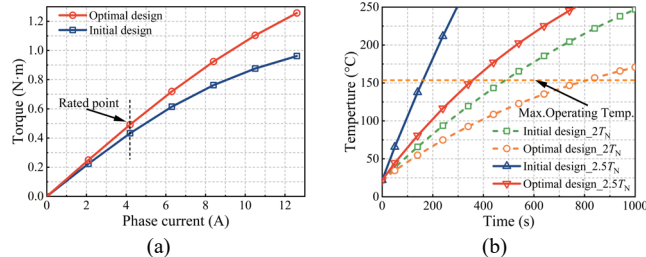


Fig. 5. (a) Torque-current curve (b) Temperature rise under different loads

TABLE II

COMPARISON OF MAXIMUM OPERATING TIME

Maximum operating time	Optimal design	Initial design
Under $2T_N$	810s	480s
Under $2.5T_N$	352s	160s

C. RSM and NSGA II

The RSM is established for the three objectives based on the Box-Behnken Design (BBD). The simulation results are analyzed using the Design-Expert software and a second-order model is used to fit the three optimization objectives. The effect of the four selected variables on the average torque is shown in Fig. 2. Due to the varying optimal ranges across individual response surfaces, conflicts arise between multiple objectives, necessitating a multi-objective optimization approach.

The NSGA-II algorithm is employed for the multi-objective optimization. Fig. 3(a) shows the optimal sets of the three optimization objectives, which shows good convergence and diversity. After some unreasonable points are eliminated, several optimized points are selected from the optimal sets based on the motor performance requirements. The optimal point is selected after comprehensive comparison.

III. ELECTROMAGNETIC PERFORMANCE ANALYSIS

Fig. 3(b) shows the average torque of the initial and optimal designs. The result indicates that the initial and optimal average torque are 433.17mNm and 492.17mNm, respectively, indicating a 13.6% increase in the optimal design. Additionally, the torque densities of the initial and optimal designs are 1.98Nm/kg and 2.17Nm/kg, respectively, representing a 9.6% improvement in the optimal design.

Fig. 4(a) presents the A-phase no-load back EMF and the corresponding total harmonic distortion (THD) of the initial and optimal designs. The fundamental amplitude of the initial and optimal back EMF are 19.62V and 21.06V, respectively, indicating a 7.3% improvement in the optimal design. Furthermore, the THD of the initial and optimal designs are 3.53% and 0.95%, respectively, which means the back EMF waveform of the optimal ST-PMVM is more sinusoidal.

The torque output versus phase current is plotted in Fig.5(a). It can be observed that the T - I curve of the optimal ST-PMVM demonstrates higher linearity, indicating enhanced overload torque output capability. Fig. 5(b) shows the temperature rise of the ST-PMVM under different load conditions ($2T_N$ and $2.5T_N$). The temperature rise rate of the optimal design is significantly slower, suggesting a longer sustainable operating time and enhanced thermal stability under short-term overload conditions. Table II summarizes the maximum operating time of the motors under different loads. Notably, the maximum operation time of the optimal ST-PMVM under $2T_N$ and $2.5T_N$ overload conditions is improved by 68.8% and 120%, respectively, further confirming the effectiveness and superiority of the multi-objective optimization method.

REFERENCES

- [1] M. Y. Metwly, C. L. Clark, J. He and B. Xie, "A Review of Robotic Arm Joint Motors and Online Health Monitoring Techniques," *IEEE Access*, vol. 12, pp. 128791-128809, August. 2024.
- [2] J. Yu and C. Liu, "Design of a Double-Stator Magnetless Vernier Machine for Direct-Drive Robotics," *IEEE Tran. Magn.*, vol. 54, no. 11, pp. 1-5, Nov. 2018.
- [3] S. L. Ho, S. Niu and W. N. Fu, "Design and Comparison of Vernier Permanent Magnet Machines," *IEEE Tran. Magn.*, vol. 47, no. 10, pp. 3280-3283, Oct. 2011.