

Minimization of eddy currents in permanent magnets of an electric machine with shape derivatives

Alessio Cesarano^{1,2}, Peter Gangl¹

¹Johann Radon Institute for Computational and Applied Mathematics, Linz, Austria

²Institute of Numerical Mathematics, Johannes Kepler University Linz, Austria

In this work we deal with the shape optimization of an electric machine considering time-dependent effects such as eddy currents. The considered electric machine is an interior permanent magnet synchronous machine and we minimize the average dissipated power due to the eddy currents in the magnets over a period of time corresponding to a rotation. Our approach is based on the computation of the shape derivative which – beside the computation of a sequence of state problems – also involves solving a sequence of adjoint equations. The challenge of this problem is related to the dependency of each one of the N adjoint equations on two different time steps, because of the finite difference used in the eddy current calculations.

Index Terms—Eddy currents, electric machines, permanent magnets, shape optimization.

I. INTRODUCTION

EDDY CURRENT losses contribute to the total losses in an electric machine, present both in ferromagnetic material and in permanent magnets. Even though the electric conductivity of the iron is higher than in rare earth materials, specific design strategies and precautions, like the use of laminated material, effectively reduce the eddy currents in the iron. Rare earth materials, on the other hand, are sensitive to heat and subject to de-magnetization. Motivated by this fact, in this work we proceed to minimize the dissipated power due to eddy current losses in the permanent magnets of an electric machine, with the use of shape derivatives and the solution a sequence of magnetostatic problems. The proposed method, when compared to evolutionary algorithms, requires much smaller computation times and allows for a larger set of admissible designs.

II. EDDY CURRENTS IN PERMANENT MAGNETS

The eddy currents induced by a variation of magnetic field can be calculated by

$$\mathbf{J}_e = -\sigma \partial_t \mathbf{A}, \quad (1)$$

in which $\mathbf{A}$ is the magnetic vector potential. The integral of $\mathbf{J}_e$ over the magnet cross section for each rotor position has to be zero, as the eddy current paths are closed in one magnet segment. This motivates the subtraction of the average value $\bar{\mathbf{J}}_e$ from the calculated current density, as suggested in [1]. The alternating current density value $\tilde{\mathbf{J}}_e$ then reads

$$\tilde{\mathbf{J}}_e = \mathbf{J}_e - \bar{\mathbf{J}}_e, \quad (2)$$

where

$$\bar{\mathbf{J}}_e = \frac{1}{\Omega_{\text{mag}}} \int_{\Omega_{\text{mag}}} \mathbf{J}_e \, d\Omega_{\text{mag}}. \quad (3)$$

In a 3-dimensional domain, eddy currents form a closed loop lying on some plane in the space. Using a 2-dimensional model at a cross section of the machine and introducing a suitable scalar function $u = u(x_1, x_2)$ as the third component of the magnetic vector potential $\mathbf{A}$, one approach to deal with eddy

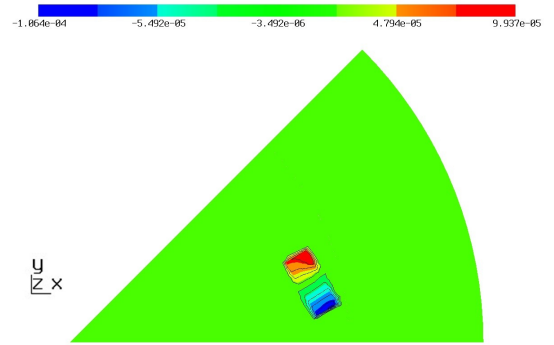


Fig. 1: Eddy currents $\tilde{\mathbf{J}}_e$ in a magnet at i -th time step. The current is assumed to be purely directed along the z -axis and the loop current to be closed at infinity.

currents is to assume that the eddy currents flow exclusively along the z -axis direction, as depicted in Fig. 1.

III. SOLUTION OF THE MAGNETOSTATIC EQUATIONS

For evaluating the eddy current losses, we solve the sequence of magnetostatics problems

$$\begin{aligned} -\nabla \cdot (\nu(x, |\nabla u|) \nabla u) &= f - \nabla \cdot (\mathbf{M}^\perp), & \text{in } D, \\ u &= 0 & \text{on } \partial D, \end{aligned} \quad (4)$$

for N different rotor positions corresponding to one electric period. In (4), ν is the magnetic reluctivity, f the impressed currents, directed along the z -axis, and $\mathbf{M}$ is the magnetization vector lying in the xy -plane. We note that the behaviour of the considered machine is periodic for each quarter and anti-periodic for each eighth, allowing for a reduction of the size of the domain. Moreover, we use the lockstep method for connecting the rotor and the stator of the machine with different angles between each other. In Fig. 2 an illustration of the solutions of the problem considered at three different rotor positions is depicted.

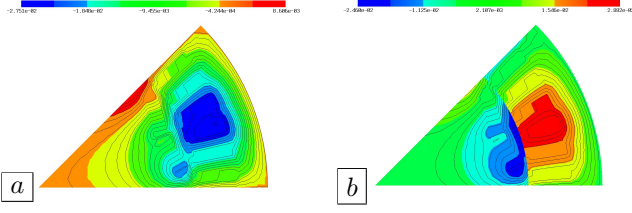


Fig. 2: Solution u , third component of the magnetic vector potential $\mathbf{A}$, to the magnetostatic problem (4) for rotor positions (a) $n = 0$ and (b) $n = N$, showing antiperiodicity.

IV. DISSIPATED POWER AND ADJOINT EQUATIONS

The dissipated power related to the eddy current losses according to Joule's law, at a fixed time step, reads

$$P = l_z \int_{\Omega} \frac{\tilde{\mathbf{J}}_e^2}{\sigma} d\Omega. \quad (5)$$

The average dissipated power is $P(\underline{\mathbf{u}}) = \frac{1}{N} \sum_{j=1}^N P_j(u_j)$ where P_j denotes the power at rotor position j . Note that we approximate the time derivative in (1) by finite differences.

In this work, we compute the shape derivative of problem (5) which is constrained by N problems of the form (4). The shape derivative can be computed in an automated way following the procedure illustrated in [2] and involves the N state solutions as well as N solutions to adjoint problems. Once the shape derivative is computed, a descent direction is extracted by solving a boundary value problem and used to deform the domain. For finding the relevant adjoint equations, let us define the Lagrangian

$$\mathcal{L}(\Omega, \underline{\mathbf{u}}, \underline{\mathbf{v}}) = P(\underline{\mathbf{u}}) + \sum_{j=1}^N \langle A_j(u_j), v_j \rangle - \langle F_j, v_j \rangle, \quad (6)$$

where A_j are the nonlinear operators corresponding to the magnetostatic problem at the j -th rotor position, while F_j , u_j and v_j are respectively the corresponding right hand side, solution and test function. The N adjoint equations are obtained by differentiating the Lagrangian with respect to u_i in the direction of test functions w_i , i.e.,

$$0 = \frac{\partial \mathcal{L}}{\partial u_i}(\Omega, \underline{\mathbf{u}}, \underline{\mathbf{v}})(w_i) = \frac{dP}{du_i}(\underline{\mathbf{u}})(w_i) + \langle A'_i(u_i)(w_i), v_i \rangle \quad (7)$$

$$\implies \langle A'_i(u_i)(w_i), p_i \rangle = -\frac{dP}{du_i}(\underline{\mathbf{u}})(w_i). \quad (8)$$

As we use order one backward Euler finite differences in time for the calculation of the dissipated power, its derivative with respect to u_i have only 2 non-zero terms out of the N terms related to the different time steps, so that

$$\frac{dP}{du_i}(\underline{\mathbf{u}})(w_i) = \frac{1}{N} \frac{dP_i(u_{i-1}, u_i)}{du_i}(w_i) + \frac{1}{N} \frac{dP_{i+1}(u_i, u_{i+1})}{du_i}(w_i). \quad (9)$$

Once the N adjoint equations are defined, the shape derivative of the cost function under scrutiny can be obtained in a rather classic fashion, see also [3].

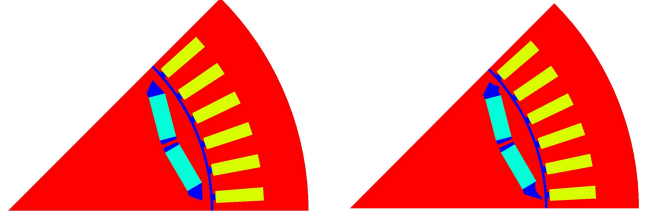


Fig. 3: Illustration of the initial design of the machine under study (left) and the optimal design (right) obtained with the proposed shape optimization algorithm. The average dissipated power due to the eddy currents in the magnets over an electric period is reduced from $P_{it0} = 0.453\text{W}$ to $P_{it21} = 0.381\text{W}$.

V. NUMERICAL RESULTS

In this section we present numerical results of the shape optimization algorithm taking into account the cost function and the adjoint equations described in the previous section. The initial design of the machine under scrutiny is depicted in Figure 3 (left), while the optimal design obtained with the proposed shape optimization algorithm is depicted in Figure 3 (right). The cost function considered, namely the dissipated power related to the eddy currents in the permanent magnets, averaged over an electric period, is reduced by roughly 20%, more precisely from $P_{it0} = 0.453\text{W}$ to $P_{it25} = 0.381\text{W}$, while the average torque increases from $T_{it0} = 575.18\text{Nm}$ to $P_{it25} = 587.74\text{Nm}$, while the torque ripple increases from 8.88% to 10.23%. The solution of N state equations requires about 12 seconds, the solution of the N adjoint equations, together with the assembling of the shape derivative requires ca. 3 seconds and the extraction of a descent direction requires, instead, ca. 2 seconds. The results are obtained with 1657 degrees of freedom and $N = 15$ rotor positions.

VI. CONCLUSIONS

The proposed work deals with the shape optimization of an interior permanent magnet synchronous electric machine, with the consideration of several rotor positions. In particular, the average dissipated power due to eddy currents in the magnets is minimized. Further developments may include the solution of the time-dependent eddy current equation, instead of the solution of a sequence of magnetostatics problems, to give a more accurate description of the time-dependent effects of the eddy currents. Moreover, the addition of a mechanical stiffness constraint may be included.

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