

Design Space Reduction in Electric Machine Optimization via Global Sensitivity Analysis

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This paper focuses on optimizing the geometric parameters and shape of a permanent magnet synchronous motor (PMSM) through isogeometric analysis, aiming to streamline the design process by reducing the parameter space. Utilizing a variance-based Global Sensitivity Analysis (GSA), we efficiently pinpoint key parameters that significantly impact motor performance and cost. By leveraging GSA outcomes, we minimize the optimization space while maintaining robust design efficacy. Additionally, this analysis provides insights into the model's behavior, which will enhance future optimization approaches.

Index Terms—Sensitivity analysis, Optimization models, Permanent magnet motors.

I. INTRODUCTION

Conventional methods in structural optimization of electric machines usually consider either the parameters or the shape separately. In [1] and [2], a combined parameter and shape optimization of a three-phase, four-pole permanent magnet synchronous motor (PMSM) is demonstrated in the context of isogeometric analysis (IGA). This model incorporates almost 50 parameters, offering extensive design flexibility and substantial improvements in performance targets. However, such comprehensive optimization is particularly challenging for electric traction applications, where multiple operational points across a drive cycle are considered.

Therefore, prior to optimization, it is beneficial to identify which parameters have the most impact on the objectives and which can be held constant without having significant design restrictions. This is accomplished through global sensitivity analysis (GSA), utilizing techniques such as the Morris Method or variance-based methods [3]. Among these, Sobol indices are particularly advantageous due to their precision and ability to reveal interactions between parameters, albeit at a higher computational cost. In this study, we employ Polynomial Chaos Expansion (PCE) as a metamodel combined with Sobol analysis, offering an efficient yet accurate method to streamline the optimization process.

We use the same model setup and optimization problem definition as defined in [1], but extend our analysis by comparing results from the original full parameter space with those from a reduced space. The reduction in parameter space is informed by identifying non-critical parameters through our global sensitivity analysis, enabling a more efficient design strategy.

II. MODEL

The same model assumptions as in [1] are used, where a four-pole PMSM is investigated using IGA in a two-

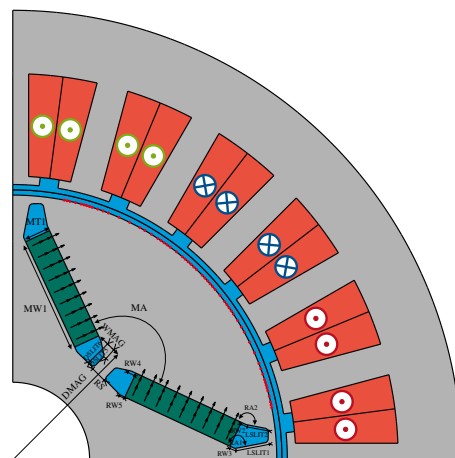


Fig. 1. Model of the PMSM showing the geometry parameters considered in the optimization. Control points which may move are highlighted in red.

dimensional MATLAB[®] framework [4]. There are 46 parameters considered in the optimization, consisting of 16 geometric parameters, one physical (the electric phase), and 29 offset values for the control points. The geometric model highlighting the geometric parameters and control points is shown in Fig. 1. There are three objectives, which are minimized during optimization with a weighted sum approach: First, the magnet area to reduce the cost. Second, a regularization term to maintain manufacturability, also called smoothness. Third, the standard deviation of the torque to reduce the torque ripple. To ensure performance, the mean torque is constrained.

III. GLOBAL SENSITIVITY ANALYSIS

GSA is employed to assess and quantify the influence of the input parameters on the uncertain output interest. Due to our lack of prior knowledge regarding the individual or combined effects of the parameters on the output, we utilize

a general variance-based method known as Sobol indices [5]. Consider a model f , mapping a vector of input parameters $\mathbf{x} = [x_1, \dots, x_i, \dots, x_M] \in \mathbb{R}^M$ to an output $Y = f(\mathbf{x}) \in \mathbb{R}$. The first-order Sobol index quantifies the impact of single factors on the output and defines the variables' importance ranking based on their exerted influence. This index is identified as

$$S_i = \frac{\mathbb{V}[\mathbb{E}[Y|x_i]]}{\mathbb{V}[Y]}. \quad (1)$$

The total-order of the Sobol index also includes information about parameter interactions. Low values indicate parameters that do not play a strong role in the computational model. This index is thus employed to select the variables that could be considered model constants, reducing the model uncertainty.

$$S_i^T = 1 - \frac{\mathbb{V}[\mathbb{E}[Y|\mathbf{x}_{\sim i}]]}{\mathbb{V}[Y]} = \frac{\mathbb{E}[\mathbb{V}[Y|\mathbf{x}_{\sim i}]]}{\mathbb{V}[Y]}. \quad (2)$$

$\mathbb{V}[\cdot]$ and $\mathbb{E}[\cdot]$ are the statistical variance and expectation operators, $[\cdot|\cdot]$ is the conditional operator, and $\mathbf{x}_{\sim i}$ identifies all input parameters except the i -th. The difference between the first and total Sobol indices thus quantifies the level of interaction between the input parameters.

The indices can be expensively computed through Monte Carlo sampling or the quasi-Monte Carlo sequences [3]. However, a polynomial chaos expansion (PCE) as a surrogate model is employed to assess the indices from the expansion coefficients [6]. The PCE $\tilde{f}(\mathbf{x})$ is formulated as

$$\tilde{f}(\mathbf{x}) = \sum_{\alpha \in \mathcal{A}} c_{\alpha} \Psi_{\alpha}(\mathbf{x}), \quad (3)$$

where $\alpha = (\alpha_1, \dots, \alpha_M)$ is a multi-index defined in $\mathcal{A} \subseteq \mathbb{Z}_0^M$, c_{α} are the coefficients, and $\Psi_{\alpha}(\mathbf{x})$ are M -dimensional polynomials, which are orthogonal to the input probability distribution functions [7].

IV. PRELIMINARY RESULTS AND DISCUSSION

The PCE model was generated using 500 sampling points drawn uniformly from the input domain as defined and used in [1] and with a maximum polynomial degree $d = 5$. The total-order Sobol indices for the magnetic area, the smoothness, and the mean torque, as well as the torque standard deviation, are shown in Fig. 2. For clarity, only the first five shape parameters, represented as control points (CPs), are included in the plot. However, the Sobol indices of the remaining CPs fall within the same range as those displayed. It can be observed that the magnet area is only sensitive to the length and width parameters of the magnet (MW1 and MT1), which is obvious and indicates the reliability of the result. Furthermore, the mean torque demonstrates a predominant sensitivity to the magnet width (MW1). The torque standard deviation is distributed over a set of parameters across the entire design space, while the smoothness is mainly sensitive to the shape parameters.

From the higher-order Sobol indices, strong interactions among all parameters can be observed for the torque standard deviation and smoothness, while the other outputs are mainly determined by single parameter sensitivities.

Parameters with low values of the total-order Sobol index can be at their nominal values. Preliminary findings indicate that

the optimized magnet area and the mean torque are comparable to those observed when utilizing the full parameter space. However, the torque standard deviation remains a challenging objective.

A comparison of the optimization using both the full and the reduced parameter ranges, along with detailed data on parameter interactions, will be presented at the conference.

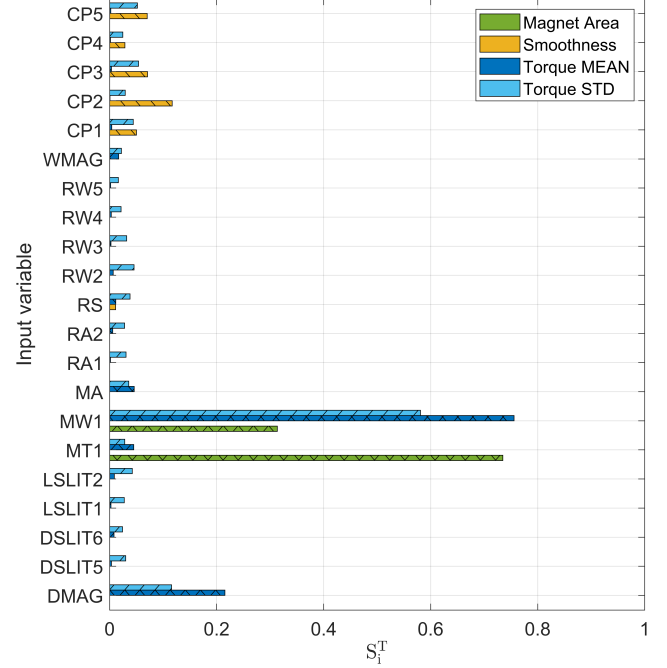


Fig. 2. Total-order Sobol indices for all geometric and 5 shape parameters and classified by the 4 objectives that are used in the optimization.

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