

Modelling non-conventional transformers with T-shaped equivalent circuits

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The electromagnetic induction phenomenon is ubiquitous and engineers are willing to exploit it for many purposes, ranging from contactless power or information transfer to energy harvesting. Simple transformer characteristics, like transformation ratio or leakage fluxes, become however elusive when the intended inductive device departs significantly from a conventional electric transformer with well-defined wound coils and a closed magnetic core. This paper shows that the T-shaped equivalent circuit is actually applicable to a rather large class of non-conventional transformers. Practical applicability conditions are provided, and the essential and irreducible limitations are reviewed.

Index Terms—Electric transformer, equivalent circuit

I. INTRODUCTION

The classical theory of electric transformers, found in any electrical engineering textbook, relies on an idealization of the magnetic coupling. This idealization is usually depicted by a pair of windings, called primary and secondary windings, embracing a closed magnetic core of high or infinite magnetic permeability, Fig. 1. Based on this picture, the theory establishes that the overall behaviour of the transformer can be modelled quite accurately with an equivalent T-shaped electric circuit. This equivalent circuit is also the main mental representation that allows engineers to anticipate the functioning of the transformer, dimension it, and control it.

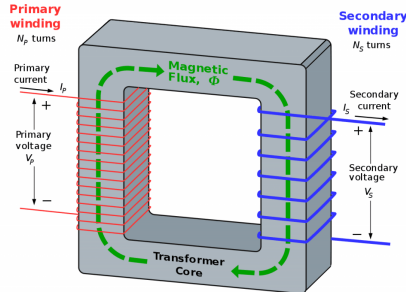


Fig. 1. Idealized theoretical set-up for the theory of transformers (Source: westline.fr).

There exist however many applications where the strong magnetic primary to secondary coupling through a high permeability magnetic core is not realized. One may think of high frequency transformers with ferrite cores of moderate magnetic permeability, cores with large airgaps in contact-free power transfer applications, or coreless planar transformers with windings etched on PCB. For such non-conventional transformers, the applicability of the equivalent T-shaped equivalent circuit becomes questionable. Moreover, the line of thoughts of the classical theory, which relies on the fact that the coupling magnetic field is tightly channeled within a magnetic core, does not generalize to coreless cases.

It is therefore hazardous for such applications to identify the lumped parameters of an equivalent circuit without searching first for a justification that the T-shaped electric circuit remains indeed a valid model. When in doubt, the ultimate solution would be to systematically resort to a finite element (FE) modelling of the device, possibly followed by a model order reduction. But FE models of transformers need in general be 3D to correctly represent leakage fluxes. This solution is therefore time-consuming and inefficient in a design phase. Moreover it does not help gaining insight and intuition about the functioning of non-conventional transformers.

The purpose of this paper is to fill a part of the gap between the idealized theory and a full explicit FE modelling of modern transformers and inductive devices. A theoretical analysis is conducted to delineate the domain of applicability of the T-shaped equivalent circuit in case of non-conventional or coreless transformers. The notions of magnetizing flux and leakage fluxes are also made accurate in such situations and their relevance to modelling and design is highlighted.

II. FROM QUADRIPOLE TO T-SHAPED CIRCUIT

Any pair of electrical conductors carrying alternating currents interact via the magnetic field, and can thus be regarded as a kind of transformer even if the conductors do not form windings with clearly identifiable numbers of turns n_1 and n_2 . As electric currents are confined within conductors, the integral quantities associated with this pair of conductors (embraced fluxes, voltages U_k , and currents I_k) are well-defined quantities. The electrical resistance R_k of the conductors can be estimated with reasonable accuracy, accounting if necessary for frequency dependent skin and proximity effects. The symmetric inductance matrix L_{ij} of the two-conductors system can then be determined by inspection, either by measurements or numerical simulations. As it is symmetric, we can let $M = L_{12} = L_{21}$. Summing up, the electrical equations

$$\begin{pmatrix} U_1 \\ -U_2 \end{pmatrix} = \begin{pmatrix} R_1 + j\omega L_{11} & -j\omega M \\ -j\omega M & R_2 + j\omega L_{22} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} \quad (1)$$

can always be identified for any pair of conductors, even if they do not look anything like a transformer. The equations (1) allow

calculating the behaviour of the 2-conductors system seen from its terminals in all situations. They have the algebraic form of a quadripole (a 2×2 matrix, complex in harmonic regime), but they cannot be represented by equivalent electrical circuit. Moreover, they fail to provide any information about the quality of the system in terms of leakage flux, for instance.

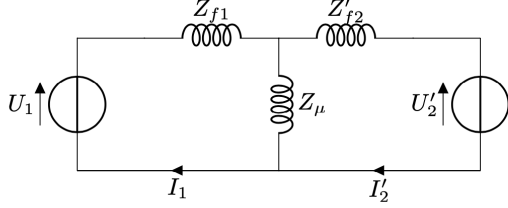


Fig. 2. T-shaped equivalent circuit of a transformer.

In order to represent the equations (1) as the solution of an equivalent electric circuit, one rewrites them under the algebraically equivalent form

$$\begin{aligned} U_1 &= R_1 + j\omega(L_{11} - \alpha M) I_1 + j\omega\alpha M \left(I_1 - \frac{I_2}{\alpha} \right) \\ -\alpha U_2 &= \alpha^2 \left[R_2 + j\omega \left(L_{22} - \frac{M}{\alpha} \right) \right] \frac{I_2}{\alpha} - j\omega\alpha M \left(I_1 - \frac{I_2}{\alpha} \right) \end{aligned} \quad (2)$$

involving now an arbitrary parameter α . The equations (2) correspond to the circuit depicted in Fig. 2 with the definitions

$$\begin{aligned} Z_{f1} &= R_1 + j\omega(L_{11} - \alpha M) = R_1 + j\omega L_{f1} \\ Z'_{f2} &= \alpha^2 (R_2 + j\omega(L_{22} - M/\alpha)) = \alpha^2 (R_2 + j\omega L_{f2}) \\ Z_\mu &= j\omega\alpha M = j\omega L_\mu \\ I'_2 &= I_2/\alpha, \quad U'_2 = \alpha U_2 \\ I_\mu &= I_1 - I'_2, \quad U_\mu = Z_\mu I_\mu. \end{aligned}$$

III. IDENTIFICATION OF THE PARAMETER α

The theoretical finding is thus that the T-shaped equivalent circuit of a transformer is only one scalar parameter away from the always valid quadripole equations (1). This is a useful result because the explicit definition of a magnetic circuit coupling primary and secondary windings is a dubious theoretical assumption in case of coreless transformers. There exists thus a whole one-parameter family of equivalent circuits that correspond to the quadripole equations (1), and it is the internal magnetic field distribution in the transformer that determines which circuit from that family, is the one that corresponds to reality.

In simple situations (Fig. 3), the parameter α can be interpreted as the transformation ratio n_1/n_2 . But in the general case, with windings that do not form simple wounded coils (think of spiral windings for instance), it cannot be determined by a simple inspection, nor measured from the accesses of the quadripole. The full paper describes a general procedure to determine α from numerical simulations. The procedure consists in determining the ratio of primary and secondary currents for which the primary and secondary windings do not exchange any coupling flux, i.e., that reproduce an ideal

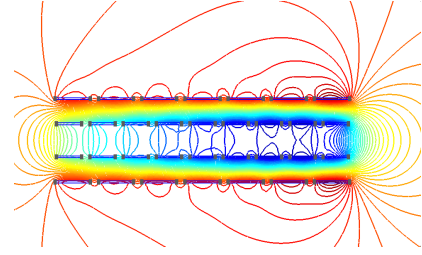


Fig. 3. Zoom-in on the magnetic flux lines around the conductors of an axisymmetric coreless planar transformer with four PCB conductor layers. The inner two layers form the primary winding with $n_1 = 2 \times 9$ turns, while the outer two layers form the secondary with $n_2 = 2 \times 7$ turns. With the imposed currents $I_1 = 1$ et $I_2 = n_1/n_2$, one verifies that no field lines crosses both a primary and a secondary conductors. This field distribution corresponds to the ideal short-circuit operation point. It determines the α parameter for this transformer.

short-circuit operation. The meaning of this condition will be illustrated with examples in the full paper, and it will also be shown that the identification procedure requires the coupling factor

$$K \equiv \frac{M}{\sqrt{L_{11}L_{22}}} \quad (3)$$

to be close to unity, which does not imply the presence of a magnetic core.

Once the value of the parameter α is determined, the elements L_μ , L_{f1} and L_{f2} of the T-shaped equivalent circuit make again sense. They restore an accurate definition for magnetizing flux and leakage fluxes in coreless transformers. They also contain valuable information about the magnetic field distribution in the transformer, that can be exploited to better characterize magnetic losses, leakage rates, and skin effect (especially in the case of PCB tracks with large width to thickness ratio). The analysis developed in this paper indicates thus that equivalent T-shape electric circuits offer an accurate model for a large class of non-conventional transformers, at least in the linear case, which can then be extended to include resonance effects [1], [2], or capacitive effects following the work of Davister [3] and references therein. Only the accurate modelling of saturation effects, magnetic losses and proximity effects make it necessary to resort to more involved FE simulations.

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