

Multi-Factor Mathematical Model and Error Analysis of Magnetic Target Localization

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Magnetic target localization is widely used in biological magnetic navigation, vehicle tracking, and buried object detection. Mathematical models and error analyses can help improve the positioning accuracy of magnetic targets. Existing research has overlooked the influence of the position and posture of a magnetic target, and lacks coupling analysis between multiple factors. Considering the complete position and posture parameters, this paper establishes a more comprehensive multi-factor mathematical model. Using this model, we identified the inaccurate area of the positioning algorithm and the compensation direction for magnetic field measurement errors, clarifying the impact of various influencing factors on positioning accuracy. The positioning experiment inside the magnetic shielding room (MSR) showed that the deviation between the simulation results and the experimental results did not exceed 2.9%, proving the accuracy of the mathematical model and error analysis.

Index Terms—Error analysis, magnetic target localization, mathematical model.

I. INTRODUCTION

Magnetic target localization is widely used in biological magnetic navigation, vehicle tracking, and buried object detection [1].

Accurate mathematical models and error analysis can help clarify the error sources in magnetic target localization and provide direction for error compensation. Higuchi *et al.* [2], Wang *et al.* [3], and Yin *et al.* [4] each proposed improved localization algorithms. Three scholars respectively made the magnetic target with unchanged posture move on a point, a line, and a surface to prove that the improved positioning algorithms enhance the positioning accuracy. However, such mathematical models ignore the influence of the position and posture of magnetic targets on positioning errors and cannot accurately evaluate positioning accuracy.

On the other hand, Higuchi *et al.* [2], Wang *et al.* [3], and Yin *et al.* [4] respectively analyzed the effects of magnetic noise, iteration times, and geomagnetic field on positioning accuracy. If the positioning accuracy cannot be accurately measured, the quantitative results of error analysis will be inaccurate. In addition, coupling analysis between multiple factors should be supplemented.

II. MULTI-FACTOR MATHEMATICAL MODEL

Considering background magnetic interference, magnetic measurement instruments, positioning algorithms, and the position and attitude of magnetic targets, we have established a multi-factor mathematical model for magnetic target localization. Due to space limitations, it will not be presented. To introduce complete position and posture parameters, this paper forms a position sphere by rotating the position vector, and an attitude sphere by rotating the magnetic moment vector. The position-posture magnetic information function $f_G(r, \theta_1, \gamma_1, \theta_2, \gamma_2)$ of the attitude sphere is proposed, as shown in Fig. 1 and (1). r and m are the modulus values of $\vec{r}$ and $\vec{m}$, respectively. θ_1 and γ_1

are the deflection and inclination angles of $\vec{r}$, respectively, while θ_2 and γ_2 are the deflection and inclination angles of $\vec{m}$. Use $f_G(r, \theta_1, \gamma_1, \theta_2, \gamma_2)$ to determine the theoretical values of $\vec{r}$ and $\vec{m}$.

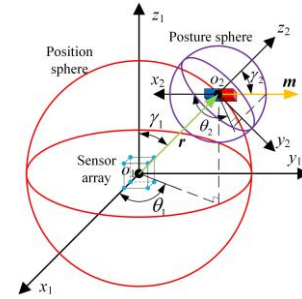


Fig. 1. Schematic diagram of position-posture magnetic information function.

$$\begin{bmatrix} x_0^t \\ y_0^t \\ z_0^t \\ m_{x0}^t \\ m_{y0}^t \\ m_{z0}^t \end{bmatrix} = f_G(r, \theta_1, \gamma_1, \theta_2, \gamma_2) = \begin{bmatrix} r \cos \theta_1 \sin \gamma_1 \\ r \sin \theta_1 \sin \gamma_1 \\ r \cos \gamma_1 \\ m \cos \theta_2 \sin \gamma_2 \\ m \sin \theta_2 \sin \gamma_2 \\ m \cos \gamma_2 \end{bmatrix} \quad (1)$$

This article proposes a diamond meshing method for spherical grids to ensure a consistent probability of occurrence for each position and posture. Unfold the spherical data into a plane to form a global area distribution, as shown in Fig. 2. N is the number of meshing layers, i and j are the meshing row and the meshing column, respectively.

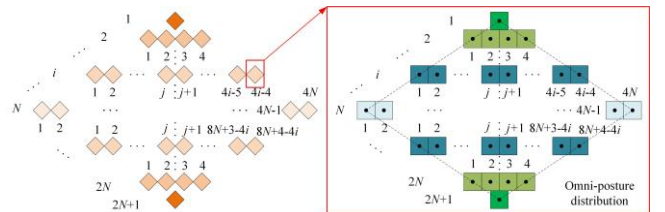


Fig. 2. Global distribution of data.

The calculation formula of the inclination γ and declination angle θ of the grid point is as follows:

$$\begin{cases} \gamma(i, j) = \frac{i-1}{2N} \cdot 180^\circ, (i = 1, 2, \dots, 2N+1) \\ \theta(i, j) = \begin{cases} 180^\circ, (i = 1 \text{ or } i = 2N+1) \\ \frac{j}{4i-4} \cdot 360^\circ, (2 \leq i \leq N+1, j = 1, 2, \dots, 4i-3) \\ \frac{j}{8N+4-4i} \cdot 360^\circ, (N+2 \leq i \leq 2N, j = 1, 2, \dots, 8N+3-4i) \end{cases} \end{cases} \quad (2)$$

III. MULTI-FACTOR ERROR ANALYSIS

The algorithm proposed by Wang *et al.* [3] is abbreviated as the WS method. Fig.3 (a) shows the global distribution of the positioning error δ of the WS method. We found that when γ_1 and γ_2 are both close to 90° , there is an inaccurate area in the WS method. Use the average value of positioning error ε_g in $f_G(r, \theta_1, \gamma_1, \theta_2, \gamma_2)$ to measure positioning accuracy. The coupling analysis of the magnetic noise standard deviation σ and magnetic field gradient g of background magnetic interference is shown in Fig. 3(b). When $\sigma < 2$ nT, the influence of σ and g on ε_g is comparable, while when $\sigma > 2$ nT, the influence of σ dominates.

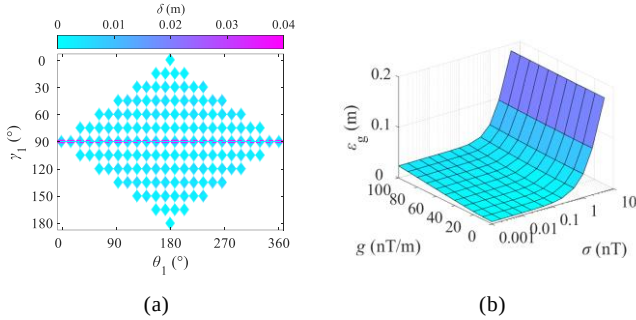


Fig. 3. Results of error analysis. (a) Global distribution of δ . (b) Coupling analysis of σ and g .

When there is a positional deviation of 1mm in the measurement point of the magnetic sensor, the positioning accuracy of the WS method will decrease from 0.0048m to 0.0075m. At this moment, the measurement point position deviation has been accurately calibrated, and the measurement error compensation method is provided by [1]. This result indicates that the error compensation method in [1] needs further improvement.

IV. EXPERIMENT OF MAGNETIC TARGET LOCALIZATION

We built a magnetic target localization platform consisting of an MSR, a magnetic sensor array, and a three-axis coil, as shown in Fig. 4. The position coordinates of the three-axis are (0.002 m, -0.300 m, 0 m), such that $\gamma_1 = 90^\circ$ and $\theta_1 = 270.38^\circ$. By controlling the current vector, $\theta_2 = 30^\circ$ and γ_2 varies within the range of 0° to 180° , with a step size of 10° . The amplitude of the coil current is 0.9 A, and the equivalent magnetic moment is $0.0316 \text{ A} \cdot \text{m}^2$.

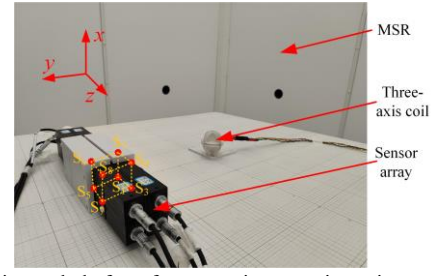


Fig. 4. Experimental platform for magnetic source inversion

The positioning error δ of the WS method is shown in Fig. 5 (a). After correction, the positioning accuracy of the WS method increased from 0.0305 m to 0.0134 m. When γ_2 approaches 90° , the WS method shows an inaccurate positioning area, which proves the accuracy of error analysis.

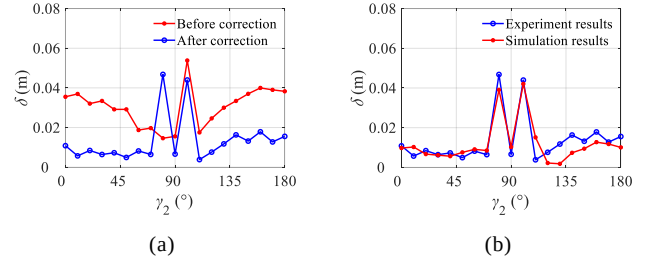


Fig. 5. Positioning error δ of the WS method. (a) Before and after correction. (b) Experiment and simulation results.

Note that the positioning accuracy of the WS method has not reached the theoretical accuracy. Measure the magnetic sensor noise when the coil is not powered on and perform Gaussian fitting on it. After Gaussian fitting, the three-axis standard deviations of the sensor noise are 0.23nT, 0.26nT, and 0.17nT, respectively. Substitute the Gaussian fitting results of sensor noise into the multi-factor mathematical model of magnetic target localization. The simulation results are taken as the average of 50 repetitions, as shown in Fig. 5 (b). In the simulation, the positioning accuracy of the WS method is 0.0131 m. The deviation between the simulation results and the experimental results does not exceed 2.9%, which proves the accuracy of the multi-factor mathematical model. Meanwhile, the results indicate that sensor noise is the main reason for the insufficient positioning accuracy in the experiment.

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