

Finite Element and Analytical Models for Predicting Eddy Current Loss in Permanent Magnet Machines

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Predicting power losses due to eddy currents in electrical conductors is an important aspect of electrical engineering research. These losses can reduce device efficiency and even cause damage in the case of excessive overheating. The issue becomes more serious when dealing with high-speed electrical machines that operate at extremely high frequencies. Permanent magnet machines designed for transportation and electric propulsion account for the majority of this category of devices. Due to the high thermal sensitivity of rare earth permanent magnets, it is critical to develop reliable and robust models for predicting eddy power loss and thus preventing permanent magnet thermal demagnetization. This work is concerned with this research issue, and a hybrid model for calculating losses in permanent magnet machines was established. It is based on combining the numerical computation of the magnetic field using the finite element approach with another analytical method for computing losses due to induced currents in permanent magnets. The model is applied for a permanent magnet synchronous machine.

Index Terms—Finite element analysis, eddy current losses, electric machines, permanent magnets.

I. INTRODUCTION

THE EDDY current loss in electric devices becomes significant as operating frequencies and conductor dimensions increase. In these cases, the theory of limited conductor resistance is no longer valid. Permanent magnet materials, notably rare earth Neodymium-Iron-Boron alloys, are commonly used in the manufacturing of high-energy performance electromagnetic devices. In addition to their strong magnetic field, these materials are regarded good electrical conductors. In the variable regime, eddy current losses arise in these conductors, which may lead to total or partial demagnetization, with negative impacts.

Considering the recent emergence of electromagnetic devices using magnets in a number of applications, researchers focused their efforts on improving their performances. Predicting and decreasing energy losses is one of the most widely researched issues in this field. This is confirmed by several scientific studies conducted on the subject. A substantial part of this research was devoted to developing reliable and robust models to analyze induced currents and the losses they generate in conductors. The finite element method is often approved as the most common calculation tool for this purpose, [1]-[2]-[3]-[4]. However, this approach becomes less appealing due to its exorbitant cost, particularly at high frequencies with a pronounced skin effect. Analytical models distinguished by simplicity and low computing cost continue to compete with the preceding model [5]-[6]-[7]. However, the simplifying assumptions on which these models are based make them less accurate than the finite element approach.

In this work, a hybridization between the two models is proposed with the aim of integrating their advantages while lowering their shortcomings. An analytical model is used to solve the electromagnetic field equation in the electric conductor, whereas a two-dimensional finite element model is used elsewhere. The two models are linked by the conditions imposed at the conductor boundaries.

The model is used to analyse and calculate the eddy current losses in a permanent magnet synchronous machine.

II. COMPUTATIONAL FRAMEWORK

A. Analytical solution of magnetic field with skin effect

Taking consideration into account the approximation of quasi-stationary states, the electromagnetic field within the magnet is governing by the Maxwell–Ampere, Maxwell–Faraday equations, Gauss’s law and magnetic relation given respectively by:

$$\vec{\nabla} \times \vec{H} = \vec{J} \quad (1)$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (2)$$

$$\vec{J} = \sigma \vec{E} \quad (3)$$

$$\vec{B} = \mu_0 \vec{H} + \vec{B}_r \quad (4)$$

with :

H, **E**, **J**, **B** are the magnetic field, electrical field, induced current density and flux density, respectively. However, σ , μ_0 and **B_r** denote the electrical conductivity, vacuum’s magnetic permeability and remanent magnetization vector.

The magnetic field equation is obtained by the combination of the above relations and it is given as:

$$\Delta \vec{H} = \sigma \mu_0 \frac{\partial \vec{H}}{\partial t} \quad (5)$$

Presuming a rectangular conductor configuration, as is shown in Fig. 1, with width w and height H_p , considered negligible in comparison to its axial length la , the magnetic field exhibits a singular component perpendicular to the conductor’s section, $H_z(x, y)$. Furthermore, consider the case of harmonic magnetodynamic regime, (5) turns to be:

$$\frac{\partial^2 \vec{H}_z}{\partial x^2} + \frac{\partial^2 \vec{H}_z}{\partial y^2} = \alpha^2 \vec{H}_z \quad (6)$$

with $\vec{H}_z$ denotes the complex notation of the magnetic field and α is given by :

$$\alpha = \sqrt{j\omega\sigma\mu_0} = \sqrt{j\frac{2}{\delta^2}}$$

where δ represents skin depth, given by : $\delta = \sqrt{2/\omega\sigma\mu_0}$

According to Stoll's work [7], if the conductor is subjected to a sinusoidal external magnetic field with a maximum value equal to H_s , the general solution of (6) is given by the following expression using the separation of variables method.

$$\bar{H}_z = H_s \left[\frac{\cosh(\alpha y)}{\cosh(\alpha \frac{a}{2})} + \sum_{n=1,3,\dots}^{\infty} Q_n \cos\left(\frac{n\pi y}{l_a}\right) \cosh\left(\sqrt{\alpha^2 + \left(\frac{n\pi}{l_a}\right)^2} x\right) \right] \quad (7)$$

with:

$$Q_n = \frac{\frac{4\alpha^2 \sin \frac{n\pi}{2}}{n\pi}}{\left(\alpha^2 + \left(\frac{n\pi}{l_a}\right)^2\right)} \frac{1}{\cosh\left(\sqrt{\alpha^2 + \left(\frac{n\pi}{l_a}\right)^2} \frac{w}{2}\right)}$$

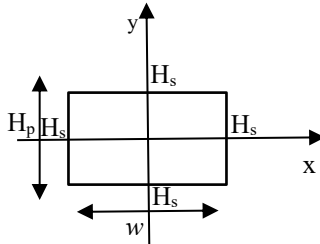


Fig. 1. Cross-section and boundary conditions of rectangular magnet.

The eddy current power losses are computed by the use of Poynting vector theory. In this case it is given by:

$$P = 2 * \text{real} \left[\frac{2H_p\alpha}{\sigma} \cdot H_s^2 \cdot \frac{w}{2} \cdot \frac{\sinh\left(\alpha \frac{H_p}{2}\right)}{\cosh\left(\alpha \frac{H_p}{2}\right)} + \sum_n \left[\frac{2\alpha^3}{\frac{H_p}{2} \left(\frac{n\pi}{H_p}\right)^2} \cdot \frac{\sqrt{\alpha^2 + \left(\frac{n\pi}{H_p}\right)^2}}{\left(\alpha^2 + \left(\frac{n\pi}{H_p}\right)^2\right)^2} \cdot \frac{\sinh\left(\left(\sqrt{\alpha^2 + \left(\frac{n\pi}{H_p}\right)^2}\right) \frac{w}{2}\right)}{\cosh\left(\left(\sqrt{\alpha^2 + \left(\frac{n\pi}{H_p}\right)^2}\right) \frac{w}{2}\right)} \right] \right] \quad (8)$$

B. 2D-Finite element model

The bi-dimensional finite element method, 2D-FEM, is used to solve the general magnetic field equation derived from the Maxwell equations using the quasi-stationary approximation and neglecting field reaction in the magnet. This general equation is given as:

$$-\nabla \times \frac{1}{\mu} \nabla \times A = J_s + \frac{1}{\mu} \nabla \times B_r \quad (9)$$

whit A is the magnetic potential vector component perpendicular to the machine's cross section along the oz axis, and J_s is the known current density parallel to A that flows through the slot.

C. Hybrid model

Using 2D-FEM to solve (9), magnetic potential values are obtained for all nodes in the domain. This allows us to determine the magnetic field values in all finite elements, from

which we can deduce the magnetic field values at the boundary nodes of the conductor.

The maximum value of the magnetic field in the analytical solution, H_s , is taken equal to the maximum value of the magnetic field on the boundaries of the conductor as result of the calculation by the 2D-FEM.

III. APPLICATION AND RESULTS

The developed model is used to investigate power eddy current loss in a three-phase synchronous machine, PMSM, equipped with surface-mounted permanent magnets. The rotor is made up of four-poles Neodymium, Iron, and Boron magnets. The stator has 24 open slots and a single-layer distributed winding. The magnet's conductivity measures $7 \times 10^5 \text{ } [\Omega/\text{m}]^{-1}$ and its remanent magnetization is equal to 1.15 [T]. The key design parameters of the studied machine are outlined in Table I. Figure 2 illustrates a comparison between finite element and hybrid calculations.

TABLE I
STUDIED MACHINE CHARACTERISTICS

Parameters	Value
Rotor yoke radius	14.5 [mm]
Air gap outer radius	20.5 [mm]
Machine outer radius	40.5 [mm]
Magnet height and span (H_p and a_p)	5 [mm] and $0.8\pi/2$ [rad]
Slot outer radius	30 [mm]

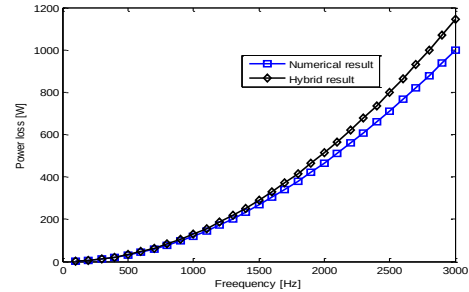


Fig. 2. Variation of power loss in magnets with Frequency.

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