

Multidomain Operator Learning Method for Electromagnetic Analyses

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The operator learning method (OLM) can directly predict the results of numerical analysis problems without going through the tedious procedure of traditional numerical methods. However, the studies on OLM so far have only focused on improving the accuracy and speed of single-domain problems. This paper proposes a multidomain OLM to solve electromagnetic field problems for practical engineering applications. The direct boundary integral equation is initially employed to solve the value and its gradient of the boundary and interface. Then, the multidomain problem is transformed into multiple single-domain problems. The OLM is employed to solve the problem of each single domain. As an example, the proposed method is used to solve the 2D electrostatic box and multi-domain electrostatic box problems, and the results are compared to the boundary element method. Finally, the proposed method is employed to solve TEAM Problem 22, and the result is compared with ANSYS/Maxwell.

Index Terms—Computational electromagnetics, operator learning, neural networks, boundary element method, numerical analysis.

I. INTRODUCTION

Machine learning has been widely applied in many industrial applications, such as autonomous driving and large language models. Attempts have also been made to accelerate the process to solve electromagnetic field problems using neural networks (NNs) [1].

After the development of several decades, traditional numerical methods, including the finite difference method, finite element method, and boundary element method (BEM), are widely employed to solve complex electromagnetic field problems in both academic and industrial designs [2]. However, the computational time is lengthy for solving some complex-shaped 3D problems, and the entire solution process needs to be repeated a great number of times for nonlinear problems.

The physics-informed neural network (PINN) [3] and the operator learning method (OLM) [4] have attracted wide attention recently. The PINN method has a heavy computational burden. The accuracy depends on the collection points because the weight in the PINN needs to be optimized for every problem to satisfy its boundary conditions (BCs) and governing equation. The OLM describes the relationship between the characteristics of partial differential equations, such as source term and shape of the compactional domain, using NNs, and thus can directly predict the results of numerical analysis problems without going through the tedious discretization, equation system solution, and post-processing procedure of traditional numerical methods. However, the OLM studies so far have only focused on improving the accuracy and speed of single-domain problems [5]. This paper proposes a multidomain OLM to solve electromagnetic field problems for engineering applications

II. PROPOSED METHODOLOGY

A. Boundary element method

The boundary element method (BEM) can be utilized for some complex shape problems because it does not need to mesh [6]. Poisson equation can be expressed as

$$\nabla^2 u = f \quad (1)$$

where u and f are the solution and the source term, respectively. The direct boundary integral equation can be used initially to compute the solution and its gradient on the boundary and interface. The matrix form of the direct boundary integral equation can be expressed as

$$\mathbf{H}_b \mathbf{u}_b = \mathbf{G}_b \mathbf{q}_b + \int_V G(x, y) f dV \quad (2)$$

where $\mathbf{u}_b$ and $\mathbf{q}_b$ are boundary elements for the Dirichlet and Neumann boundary conditions, respectively. $\mathbf{H}_b$ is the coefficient matrix for the normal potential derivative, and $\mathbf{G}_b$ the coefficient matrix of potential. $G(x, y)$ can be calculated as

$$G(x, y) = -\frac{1}{2\pi} \log r \quad (3)$$

where r is the distance between the source point and the field point. Then, the value of interior points $\mathbf{u}_i$ can be obtained by

$$\mathbf{u}_i = \mathbf{G}_i \mathbf{q}_b - \mathbf{H}_i \mathbf{u}_b + \int_V G(x, y) f dV \quad (4)$$

B. Multidomain Operator Learning Method

When the side lengths of the computational domain are proportionally expanded for a rectangular electrostatic field with fixed BCs and an unchanged shape, the values at points with the same relative positions remain unchanged. When the BCs are multiplied by a factor, the solution of the points in the same position is also multiplied by this factor [7]. Therefore, the length of the compactional domain and the BCs can be initially transformed to $[0, 1]$ and $[-1, 1]$, which is defined as a standard problem. An NN can be trained to predict all the combinations of input parameters, including the relative permittivity, ϵ_1 , the aspect ratio of the computational domain, W_c , and the BCs of the four edges, $\mathbf{B}_1$, $\mathbf{B}_2$, $\mathbf{B}_3$, and $\mathbf{B}_4$, as shown in Fig. 1. For given input data, the initial problem can be transformed by solving the standard problem. Finally, the solution to a given problem can be obtained in the output of the NN.

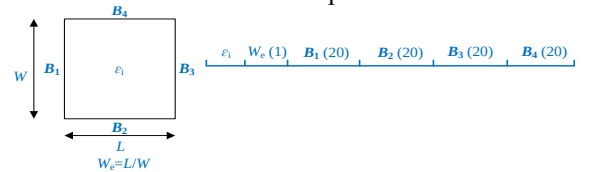


Fig. 1. Input of neural network.

Fig. 2 shows a flowchart of the proposed OLM for multidomain problems. It includes six main steps.

Step 1: Partition the solution domain. The whole computational domain is divided into different subdomains according to their shape and governing equation.

Step 2: Calculate the interface's value by direct boundary integral equation (2). The direct boundary integral equation of different subdomains is coupled and solved simultaneously. The BC of each subdomain, including the Dirichlet and Neumann

BCs, is solved.

Step 3: Transform the domain into a standard domain. The length and width of the solution area are divided by the length of the subarea to transform the boundary to $[0,1]$.

Step 4: Transform BC to $[-1,1]$. The BC of the subdomain is divided by the maximum absolute value of the element in BC, which is defined as $\text{abs}(BC_{\max})$.

Step 5: Solve the standard area for given input to NN: ε_i , W_e , and BC_1 of the standard area.

Step 6: Transform to the initial domain. The final result (FR) of the subdomain can be obtained from the result of the standard domain ($RoSD$) and $\text{abs}(BC_{\max})$.

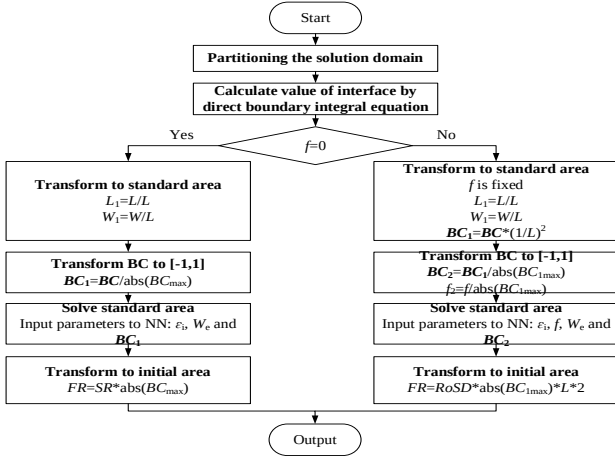


Fig. 2. Flowchart of proposed multidomain OLM.

III. 2D ELECTROSTATIC PROBLEM

As an example, the Laplace equation describing electrostatic field problems for given BCs is solved using the proposed method and compared with BEM:

$$-\nabla \cdot (\varepsilon_i \cdot \nabla u(x, y)) = 0 \quad (5)$$

where $u(x, y)$ is the electric potential at field point (x, y) . ε_i is the relative dielectric constant for the solution domain.

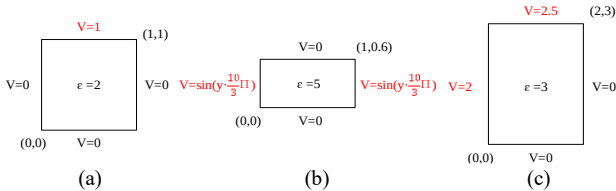


Fig. 3. Electrostatic problems. (a) Problem 1, (b) Problem 2, (c) Problem 3.

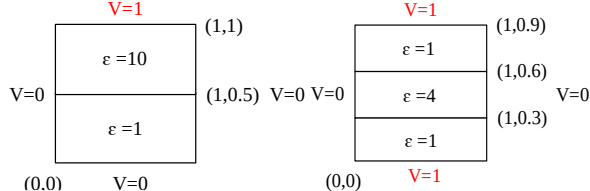


Fig. 4. Multidomain electrostatic problems. (a) Problem 4, (b) Problem 5.

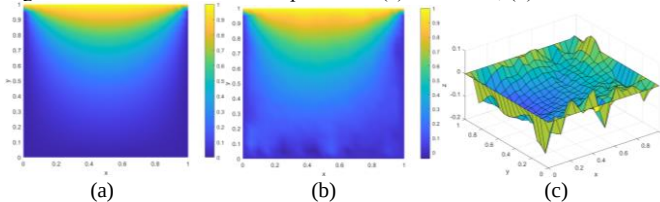


Fig. 5. Results of problem 1: (a) BEM, (b) OLM, (c) Error.

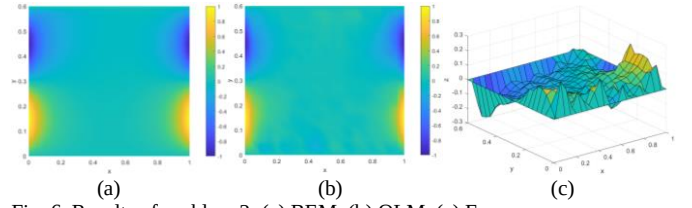


Fig. 6. Results of problem 2: (a) BEM, (b) OLM, (c) Error.

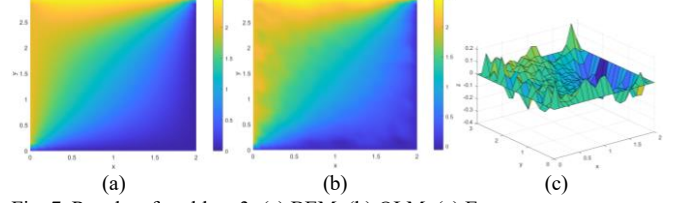


Fig. 7. Results of problem 3: (a) BEM, (b) OLM, (c) Error.

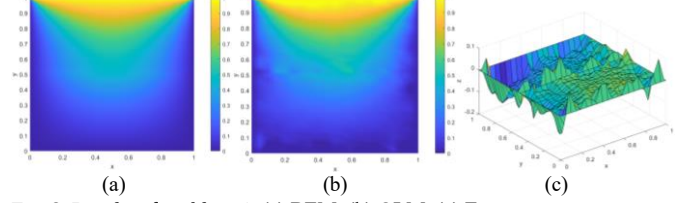


Fig. 8. Results of problem 4: (a) BEM, (b) OLM, (c) Error.

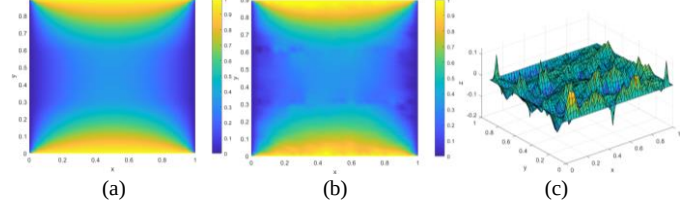


Fig. 9. Results of problem 5: (a) BEM, (b) OLM, (c) Error.

The results of the OLM for Problems 1-5 are presented in Figs. 5-9, respectively. The maximum absolute errors between the BEM and OLM are 0.1655, 0.2380, 0.2706, 0.1453, and 0.1290 V for Problems 1-5, respectively, showing the effectiveness of the proposed method.

IV. TEAM PROBLEM 22

The results of the TEAM 22 problem will be presented in detail in the full paper.

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