

Improved Computational Speed of an Analytical Differential Coil Model for Eddy Current Testing of Coated Steel Sheets Using Series Expansion

Martin Koll¹, Daniel Wöckinger¹, Christoph Dobler¹, Gerd Bramerdorfer¹,
Stefan Schuster², Stefan Scheiblhofer², Norbert Gstötenbauer² and Johann Reisinger²

¹Institute of Electric Drives and Power Electronics, Johannes Kepler University Linz, Altenbergerstraße 69, Linz, Austria

²voestalpine Stahl GmbH, voestalpine-Straße 3, Linz, Austria

Non-destructive testing for electrically conductive objects is usually carried out by impedance measurements using eddy current coils. Analytical integral impedance models for these type of problems are well-known in the literature. These integral models typically have a long calculation time. Therefore, series expansion models have been developed to speed up the calculation time by using the so-called Truncated Region Eigenfunction Expansion. This method is typically used to model single coils which consist of one single winding to generate and measure electromagnetic fields. Nevertheless, using a separate excitation and measuring system in an eddy current testing sensor is often advantageous. This paper applies the truncated region eigenfunction expansion method to a cylindrical excitation coil involving two vertically and coaxially symmetrical pickup coils placed next to a coated sheet metal and presents a novel series expansion formulation. Furthermore, the advantages of the presented analytical model in terms of calculation time improvement and their applications with regard to the parameter estimation of coated steel sheets are shown. In addition, the new series expansion model offers much better error control because the number of series elements can be specified.

Index Terms—Eddy current testing, thickness measurement, parameter estimation, analytical models

I. INTRODUCTION

EDDY current testing (ECT) is widely used for non-destructive testing of electrically conductive objects in industry. In this paper, the focus is set on the material and geometric parameter determination of metal sheets with thin, electrically conductive and diamagnetic coatings on electrically conductive and ferromagnetic steel substrates. An eddy current coil is typically used to determine the impedance of the arrangement, leading to the development of advanced analytical models in the past. Since an analytical inversion of the analytical integral models presented in [2] is not possible, unknown sheet metal and coating parameters, e.g., coating thickness, coating conductivity, or substrate parameters, must be identified using estimation methods. Due to the relatively long forward calculation time of the integral models, real-time capability cannot be achieved in many processes, e.g., to facilitate an inline coating thickness estimation. For this reason, it is essential to speed up the computing time of these models. In the integral model approaches [2], the magnetic problem boundaries are infinite. This leads to an integral with an upper limit of infinity when solving the differential equations. In the approach presented in [3], the solution region is truncated in one dimension. This method is called Truncated Region Eigenfunction Expansion (TREE). This approach leads to a finite number of eigenfunctions. The corresponding solution is composed as a combination of these eigenfunctions and thus results in a sum with a finite number of elements instead of the infinite integral. In this paper, as in [3], a rotationally symmetric eddy current problem is assumed and the radial expansion is limited. However, the solutions of the impedance via the TREE method in literature are typically given for

single ECT coils, but it is often advisable to build the sensor system from an excitation coil and one or more pick-up coils, and to evaluate the mutual impedance ζ of the arrangement. This paper aims to apply the TREE method to the differential coil presented in [2], and to compare it to the integral model with regard to various aspects, e.g., computational time and accuracy. In the original publication of the integral model, this coil is referred to as a reflection coil. The described coil setup is shown in Fig. 1. It yields in a galvanic isolation of the input

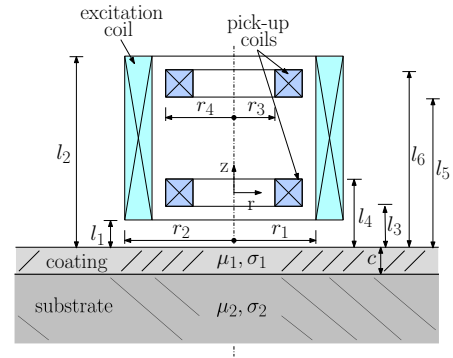


Fig. 1. Differential coil setup and selected parametrization.

and output of the sensor. The typically temperature-dependent ohmic resistance of the excitation coil is therefore no longer present in the measuring voltage at the secondary-side. Since the two pick-up coils are wound in opposite directions and connected in series the differential voltage is zero if no sheet metal is measured.

II. METHODOLOGY

Starting from the analytical integral model of the differential coil known in the literature, a novel series expansion model

for this differential coil setup is presented based on the TREE method in [3]. Furthermore, the validity and accuracy of the new series expansion modeling approach are analyzed for different coil geometries and pick-up coil positions. Constraints on the coil geometry are defined to ensure the accuracy of the series expansion. Additionally, different numbers of series elements and different problem bounds radii are used and these results are compared with those of the integral model regarding each derived mutual impedance. Typical material parameters to be expected in practice are used for this purpose. Another important aspect investigated is the achieved reduction of computing time, when compared to the integral model. Since the application of this model type is concerned with the model-based parameter estimation of the sheet metal and liftoff, the improved computational speed and accuracy are analyzed and highlighted.

III. FUNDAMENTALS

The underlying differential equation of the vector potential A for a rotationally symmetric problem including electrically and magnetically conductive material can be derived from Maxwell's equations [1]

$$\frac{\partial^2 A}{\partial r^2} + \frac{1}{r} \frac{\partial A}{\partial r} + \frac{\partial^2 A}{\partial z^2} - \frac{A}{r^2} + \omega^2 \mu_0 \mu_i \epsilon_i A - j\omega \mu_0 \mu_i \sigma_i A = 0, \quad (1)$$

where r and z are the cylindrical coordinates of a point under consideration, $\omega = 2\pi f$ is the electrical angular frequency, μ_i , ϵ_i , and σ_i are the relative permeability, the relative permittivity and the conductivity of the material. The index i indicates the different material layers, as shown in Fig. 1. The general solution of this differential Equation (1) has been derived in [2] and is given as

$$A(r, z) = \int_0^\infty (C_1(\alpha) e^{\alpha_i z} + C_2(\alpha) e^{-\alpha_i z}) \times (C_3(\alpha) J_1(\alpha r) + C_4(\alpha) Y_1(\alpha r)) d\alpha, \quad (2)$$

with $J_1(x)$ a Bessel function of first kind and order and $Y_1(x)$ is a Bessel function of second kind and first order. The unknown factors C_1 to C_4 must be determined by boundary and transition conditions between the individual model regions. Furthermore, the characteristic number of the problem

$$\alpha_i = \sqrt{\alpha + j\omega \mu_0 \mu_i \sigma_i} \quad (3)$$

is required. A step-by-step integral solution to this type of eddy current differential equation system was provided in [2].

IV. OUTLOOK - SERIES EXPANSION MODEL

The integral model assumes an infinite radial and axial expansion of the problem. In comparison, the TREE method introduces an additional magnetic boundary condition at the radial distance r_0 . This homogeneous Dirichlet boundary condition $A(r_0, z) = 0$ results in the magnetic vector potential solution of the differential equation becoming a sum instead of an integral compared to Equation (2). As shown in [3] it can be defined as

$$A(r, z) = \sum_{k=1}^{\infty} J_1(\gamma_k r) (C_k e^{-\delta_{ki} z} + D_k e^{\delta_{ki} z}) \quad (4)$$

where γ_k and δ_{ki} are the characteristic numbers of the new problem. The unknown constants C_k and D_k can be calculated using the interface conditions between the individual problem layers illustrated in Fig. 1. The discrete eigenvalues of the problem which fulfill vanishing field at distance r_0 can be calculated via [3]

$$J_1(\gamma_k r_0) = 0 \text{ or } J_1(x_k) = 0 \text{ and } \gamma_k = \frac{x_k}{r_0}. \quad (5)$$

From the solution of the vector potential A , the voltage induced in the pick-up coils and thus the mutual impedance ζ can be calculated. In theory, the sum in Equation (4) is unbounded. However, as visible in Fig. 2, it is sufficient to use a relatively small number of series elements N_s to obtain a good approximation of the solution. Fig. 2 shows the mutual impedance for a selected coil geometry and sheet metal properties as a complex frequency locus with a different number of series elements N_s . In the final paper, this novel series expansion model will be derived in detail and analyzed regarding various aspects, e.g., computational speed, accuracy, and geometry dependence. Compared to the integral model, the forward computing time can be improved by a factor of over 200. The selectable number of series elements N_s leads to enhanced error control. Furthermore, the very accurate and fast estimability of the sheet metal parameters, e.g., coating thickness c , liftoff L , and the relative substrate permeability μ_2 , can be confirmed by applying a model-based estimator. This allows real-time capability, which is required for fast industrial processes. This work has been supported by the LCM

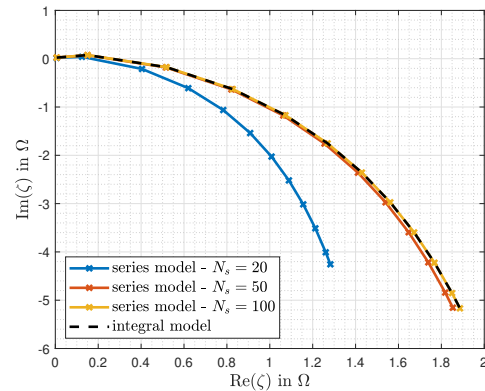


Fig. 2. Studies on different numbers of series elements N_s and their effects on the frequency locus.

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