

A New Anhysteresis Curve for Magnetic Materials

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This paper introduces a new anhysteresis curve for electromagnetic simulations such as transformers, rotating machines, or magnetic devices. Unlike existing anhysteresis curves, this curve has several advantages, including continuous differentiability, smoothness for numerical solvers, and characterization using four parameters, each responsible for a specific magnetization region. The new anhysteresis curve is validated by fitting various magnetic materials. Its incorporation into an energy-based hysteresis model is also discussed briefly.

Index Terms—Electromagnetic measurements, Electromagnetic modeling, Magnetic materials, Magnetic hysteresis, Magnetization processes

I. INTRODUCTION

ONE crucial element in modelling magnetic materials is the constitutive relation itself. The anhysteresis curve, introducing a connection between the applied magnetic field intensity h and the magnetization m , can describe this monotonically increasing constitutive relation using analytical functions, such as the inverse tangent, the (double) *Langevin* function [1], or other functions with saturation-like behaviour. A comparison of several analytical anhysteresis functions can be found in [2]. However, most of these functions have the disadvantage that usually only one or two parameters can be adjusted, limiting the degrees of freedom for fitting to dedicated measurements. One exception is the double *Langevin* function, where four parameters can be adjusted. A minor disadvantage of this function is the discontinuity for zero input, which can be overcome by forcing the value to zero. This zero forcing fails for the function's first derivative since the slope is non-zero. The first derivative is potentially necessary in a numerical solution scheme. Spline functions can also be used for depicting the anhysteresis curve [3]; however, since measurements are always corrupted with noise and other uncertainties, a spline fit might result in non-monotonicity of the resulting anhysteresis curve, which possibly, becomes a problem in numerical solvers that use the first derivative.

This paper introduces a new anhysteresis curve originating from depicting slip-dependent lateral tire forces in vehicle dynamics. The presented curve is a smooth and continuously differentiable function characterized by four coefficients, each responsible for a specific region of the anhysteresis curve—a straightforward manual fit is proposed besides a numerical optimization. The anhysteresis curve is then applied to various magnetic materials, whereas the accuracy is compared to existing functions (double *Langevin* / inverse tangent / hyperbolic tangent). Incorporating the anhysteresis curve into an energy-based hysteresis model [4] is discussed briefly.

II. THE ANHYSTERESIS CURVE

The tire and vehicle dynamics book of *Hans Pacejka* [5] describes slip-dependent lateral tire forces by an empiri-

cal formula—the so-called *magic formula of Pacejka*—which reads in general as

$$f(x) = c_1 \sin(c_2 \arctan(c_3 x - c_4 (c_5 x - \arctan(c_6 x))))), \quad (1)$$

with $c_1, \dots, c_6 \in \mathbb{R}$. By choosing these coefficients arbitrarily, the function may reveal a saddle point and a global minimum and maximum. However, a modification of (1) results in a candidate for an anhysteresis function with saturation-like behavior as

$$m_{\text{prop}}(h) = m_{\text{sat}} \sin(\arctan(a h + b \arctan(c h))), \quad (2)$$

with m_{sat} the saturation magnetization, and (a, b, c) three adjustable parameters.

To guarantee strict monotonicity of (2), the parameters $(m_{\text{sat}}, a, b, c)$ must be either all positive, or the inequalities

$$m_{\text{sat}} \geq 0, \quad a \geq 0, \quad bc \geq 0 \quad (3)$$

must be fulfilled during a manual fit or a numerical optimization procedure using measurement data. The first two inequalities in (3) ensure a positive saturation magnetization and a strictly increasing linear contribution in the outer inverse tangent's argument in (2), respectively. The third inequality in (3) ensures a strictly monotonic increasing inner inverse tangent in (2). An easy manual fit to measurements is proposed since each of these four parameters is responsible for one characteristic influence on the shape of the anhysteresis curve, as Figure 1 depicts.

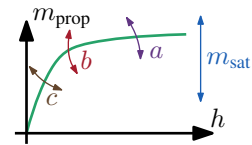


Fig. 1. Influence of the individual parameters on the proposed anhysteresis curve in (2)

Thus, a , b , and c are responsible for the curvature after the knee, the height of the knee, and the curvature before the knee in the origin, respectively.

III. NUMERICAL FITTING PROCEDURE AND EXAMPLES

Besides a manual adjustment, a numerical fitting procedure to determine the coefficients (m_{sat}, a, b, c) in (2) can be set up, e.g., by minimizing the L_2 -norm $\|\cdot\|_2$ of the error vector between a measured magnetization vector m_{meas} and (2), reading as

$$\begin{aligned} \arg \min_{m_{\text{sat}}, a, b, c} & \|m_{\text{prop}}(h_{\text{meas}}) - m_{\text{meas}}\|_2, \\ \text{subject to (3) or } & (m_{\text{sat}}, a, b, c) \geq 0, \end{aligned} \quad (4)$$

with h_{meas} the measured magnetic field intensity vector in correspondence to m_{meas} .

To demonstrate the proposed anhysteresis curve, (2) is fitted to various magnetic materials (three silicon steel sheets from different transformers and armco iron) using (4) and the constraints from (3). The results are depicted in Figure 2 and the corresponding obtained parameters are listed in Table I.

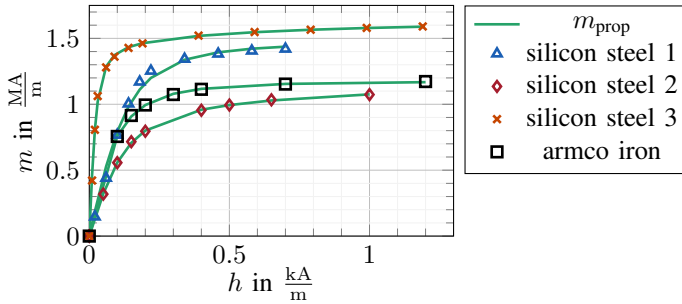


Fig. 2. Various magnetic materials used for fitting the proposed anhysteresis curve (2) using MATLAB's `fmincon`

TABLE I
PARAMETERS OF THE FITTED ANHYSTERESIS CURVES TO VARIOUS MAGNETIC MATERIALS

	m_{sat}	a	b	c
	$\frac{\text{MA}}{\text{m}}$	$\frac{\text{mM}}{\text{A}}$	-	$\frac{\text{mA}}{\text{m}}$
silicon steel 1	1.47	6.39	455	0.15
silicon steel 2	1.16	1.25	0.83	5.56
silicon steel 3	1.64	1.69	-1.23	-23
armco iron	1.19	3.86	-1.05	-4.49

Compared to existing analytical functions such as the double *Langevin* function, the simple inverse tangent, and the hyperbolic tangent, the proposed function shows the smallest error ($< 5\%$ mean rel. deviation) to the measurement points in almost all of the fitted cases from Figure 2 using the optimization procedure from (4).

IV. INCORPORATION OF THE ANHYSTERESIS CURVE IN AN ENERGY-BASED HYSTERESIS MODEL

The proposed analytical anhysteresis function (2) can be incorporated into a hysteresis model, which is chosen here as an energy-based hysteresis model based on dry friction-like pinning [4, 6]. The solution strategy therein to obtain the magnetization as a function of the exciting field intensity is the

Newton-Raphson method, minimizing dedicated energy functionals; see [6] for further details. Therein, the first derivative of (2) is necessary, which can be stated analytically as

$$\frac{dm_{\text{prop}}}{dh} = m_{\text{sat}} \cos(A) \frac{1}{1+B^2} C, \quad (5)$$

with the corresponding terms

$$\begin{aligned} A &= \arctan(a h + b \arctan(ch)), \\ B &= a h + b \arctan(ch), \\ C &= a + \frac{bc}{1+(ch)^2}. \end{aligned}$$

This first derivative is—compared to the double *Langevin* function—continuous and straightforward to implement.

V. CONCLUSION

A new anhysteresis curve originating from tire and vehicle dynamics is presented. This smooth analytic and continuous differentiable function is characterized by four parameters, each responsible for a dedicated region influencing the shape of the curve. The numerical optimization procedure to determine these parameters is demonstrated on materials such as steel sheets and armco iron. The proposed anhysteresis function shows the lowest error for almost all materials compared to existing functions. Incorporating the curve into an energy-based hysteresis model is briefly discussed and can be done straightforwardly with the discussed first derivative. The full contribution discusses the accuracy of the proposed anhysteresis function, a comparison to existing functions in more detail, and the incorporation into the finite element method.

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