

Identification of the Shape Function of a Play Model Using Parametric Polynomial Approximation

H. Morishita¹, K. Sugahara¹, *Member, IEEE*, Y. Hane² *Member IEEE*, and A. Ahagon³ *Member, IEEE*

¹Faculty of Science and Engineering, Kindai University, Higashiosaka, 577-8502 Japan, ksugahar@kindai.ac.jp

²Faculty of Science and Engineering, Toyo University, Kawagoe, 350-8585 Japan, hane@toyo.jp

³JMAG Business Company, JSOL, Osaka, 550-0001 Japan, ahagon.akira@jsol.co.jp

This paper proposes a simple interpolation method employing parametric polynomial approximation to identify the shape function of a play model. Among various hysteresis models, the play model provides relatively high accuracy; however, its precision is heavily influenced by the number of play hysterons, which depends on the number of measured hysteresis loops. Achieving this level of accuracy in applications such as accelerators requires both appropriate hysteresis loop divisions and approximately 1,000 hysteresis loops, which makes such extensive measurements highly demanding. This effort is generally unnecessary for standard applications like motors but is critical for high-precision applications. To address this issue, this study proposes a method for interpolating and generating a large number of symmetric hysteresis loops from a small number of measurement data. The proposed method successfully reduces the burden of data collection while maintaining high accuracy through the interpolation of hysteresis loops with equidistant maximum values. Normalization effects brought this success. The proposed approach is expected to have future applications in the characterization and design of accelerator magnets.

Index Terms—Hysteresis, interpolation, play model, rational polynomials.

I. INTRODUCTION

The play model is mathematically equivalent to the Preisach model, which is widely employed for numerically reproducing complex hysteresis behavior [1]. It further reduces computational costs. The play model is widely used because it can represent various hysteresis loops by combining multiple play hysterons and shape functions. Hysterons are the fundamental structural elements that respond while storing their internal states and play an important role in describing magnetic hysteresis phenomena [2]. The shape functions of the play model are identified with an evenly spaced distribution of B - H loops, employing equidistant intervals of the maximum magnetic flux density $B_{\max}$ [3]. However, measuring numerous loops while strictly controlling this distribution in experimental data is challenging and time-consuming. Previous attempts have utilized Gaussian process regression [4] and B-spline interpolation [5] to address this issue. This paper proposes a simpler interpolation method based on parametric polynomial approximation.

The method applies parametric polynomial approximation to interpolate the measured hysteresis loops. Consequently, the magnetic field H can be expressed as a function of $B_{\max}$ and B . It enables interpolation of hysteresis loops with equally spaced $B_{\max}$ (equidistant $\Delta B_{\max}$) even when the measured data has non-equidistant $\Delta B_{\max}$. It also facilitates the identification of shape functions of play models based on a relatively small dataset.

The findings of this study can be applied to the analysis of hysteresis characteristics in accelerator magnetics. For instance, in accelerators, $\Delta B_{\max}$ with 0.1% accuracy is necessary, requiring approximately 1,000 major loops.

II. HYSTERESIS INTERPOLATION

Fig. 1 illustrates a flowchart of the proposed method for obtaining the interpolated hysteresis loops from measured hysteresis loops. $B_{\max}$ and $B_{\min}$ denote the maximum and minimum flux density of each loop, respectively. $H_{\max}$ and $H_{\min}$ denote the maximum and minimum magnetic field of each loop, respectively.

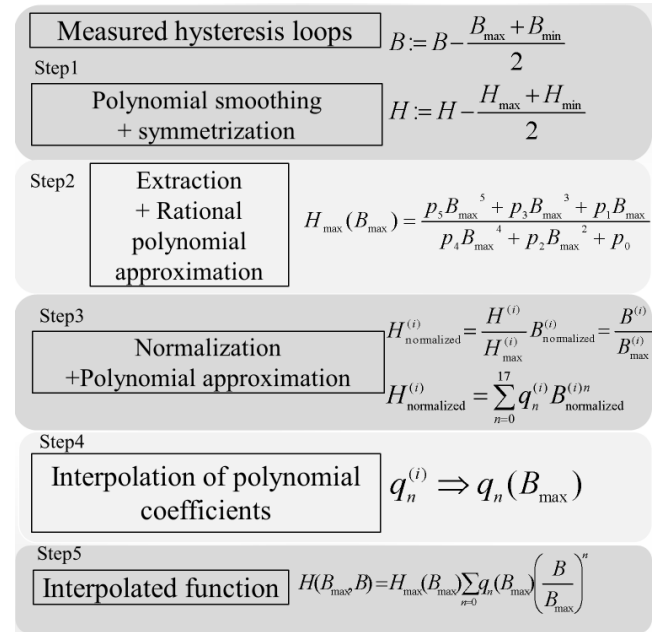


Fig. 1. Flowchart of the hysteresis interpolation

Initially, in Step 1, polynomials fittings are employed to smooth the measured magnetic hysteresis curves, which often contain noise and irregular fluctuations, as shown in Fig. 2. This step removes these fluctuations, and the hysteresis curves are subsequently symmetrized. Next, in Step 2, curve fitting is

using a rational polynomial, with both parts as 5th-degree polynomials, selected for accuracy, stability, and consistency. p_n represents the set of coefficients that constitute the numerator and denominator of a rational polynomial function. To reflect the odd symmetry of magnetic hysteresis curves, the numerator is set as an odd function and the denominator as an even function. The denominator's constant term is fixed at $p_0 = 1$ to ensure proper scaling and enhance convergence, otherwise, p has excessive degrees of freedom. This approach faithfully reproduces the curve's physical characteristics while suppressing oscillations common in standard polynomial fitting. Although poles may appear between measured points, this can be avoided by adjusting the initial values of the optimization, resulting in a smooth, monotonically increasing function. In Step 3, by normalizing each B - H curve with different $B_{\max}^{(i)}$ and $H_{\max}^{(i)}$ so that all maxima are scaled to unity, the entire set of loops can be expressed as a parametric family of functions. In Step 4, polynomial coefficients q_n are interpolated as functions of $B_{\max}$, creating a continuous representation. In Step 5, these coefficients are used to derive a function that computes H for any B and $B_{\max}$.

III. VERIFICATION OF THE PROPOSED METHOD

To validate the method proposed in Section II, we applied it to a measured ring core made of non-oriented silicon steel (35A360) and the results are shown in Figs. 2 to 4. The measured hysteresis loops were targeted at 0.05 T increments within the range of 0.05 T to 1.5 T. Fig. 2 shows the smoothed and symmetrized hysteresis loops in the range of 0.05 T to 0.25 T, illustrating the reduction in measurement noise. Fig. 3 shows the normalized B - H curves obtained in Step 3, demonstrating a parametric set of functions that continuously transform with respect to $B_{\max}$.

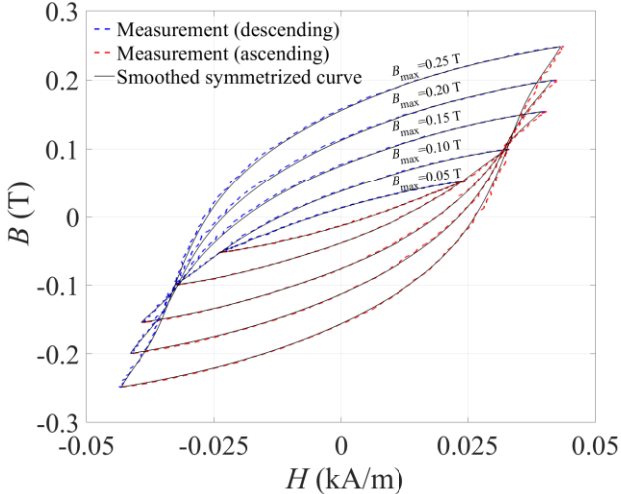


Fig. 2. Smoothing and symmetrization within the range of 0.05T to 0.25T

In Step 5, as shown in Fig. 4, $\Delta B_{\max}$ is set to 0.02 T. The red lines represent ascending, while the blue lines represent descending. The proposed method enables the interpolation of the hysteresis loops with arbitrary $\Delta B_{\max}$. The input measured hysteresis loops do not have to have equidistant intervals of $\Delta B_{\max}$. The only requirement of the proposed method is that the measured hysteresis loops are free of noise, which can also be achieved through careful smoothing.

The success of this relatively simple method is primarily attributed to the significant effect of normalization. This normalization method may also be effective for the method using Gaussian process regression proposed in [3], but this has not yet been tested. Since the results of Gaussian process regression depend on the kernel function and its hyperparameters, the methods using parametric polynomials are simpler. In the future, this approach will be applied to the study of accelerator magnets.

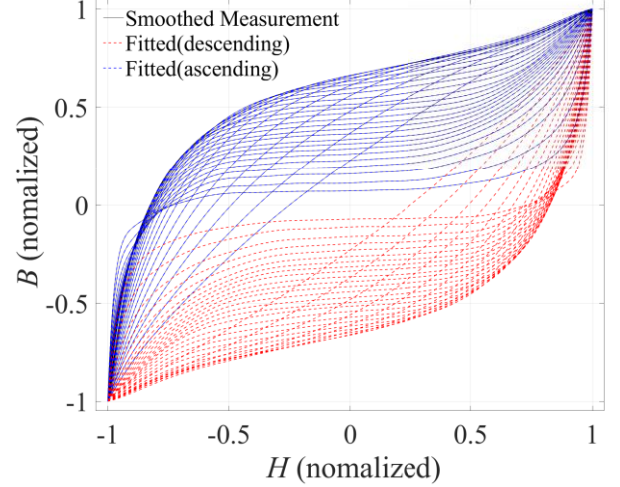


Fig. 3. Normalized hysteresis loop with both B and H ranging from -1 to 1

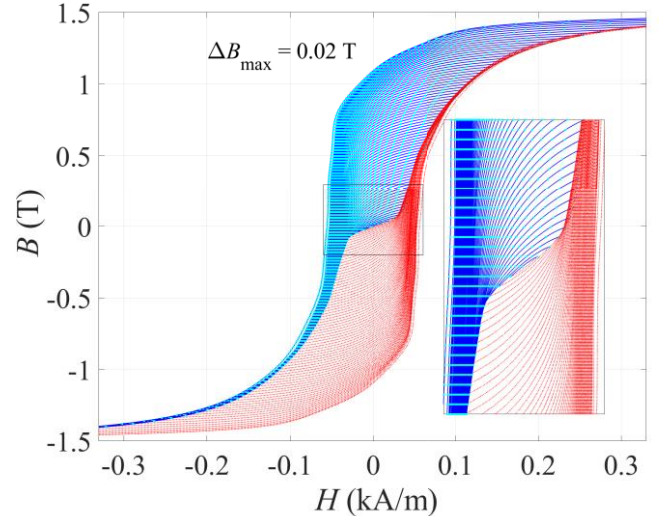


Fig. 4. Interpolated hysteresis loops with equally spaced B and $B_{\max}$

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