

Efficient Evaluation of an Inverse Energy-Based Magnetic Vector Hysteresis Model

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The energy-based vector hysteresis model of Francois-Lavet et al. establishes an implicit relation between magnetic fields and fluxes via internal polarizations each of which is determined by a convex but non-smooth minimization problem. As shown in a recent contribution by the authors, the inverse relation can be defined in a similar manner. The computation of the internal variables however now leads to a coupled convex and non-smooth minimization problem whose dimension depends on the number of the internal variables. We show that this problem can be solved efficiently. After regularization of the non-smooth terms a Newton method with line search can be applied. By introducing the total polarization as an additional internal unknown, the Newton step can be reduced to solving independent small linear systems for each internal variable. The computational complexity for evaluating the inverse hysteresis operator thus essentially reduces to the same as that for computing the forward hysteresis operator. The theoretical results are illustrated by comparing the computation times for some standard benchmark problems.

Index Terms—magnetic vector hysteresis, inverse hysteresis operator, regularization, Newton method

I. FORWARD HYSTERESIS OPERATOR

THE constitutive models for ferromagnetic materials are typically based on the fundamental identity

$$\mathbf{B} = \mu_0 \mathbf{H} + \mathbf{J}. \quad (1)$$

Here $\mathbf{B}$ denotes the magnetic induction, $\mathbf{H}$ the magnetic field intensity, and $\mathbf{J}$ the total magnetic polarization, which describes an internal state of the system. In the hysteresis model of [1] the total magnetic polarization is split as $\mathbf{J} = \sum_{k=1}^K \mathbf{J}_k$ and the partial polarizations $\mathbf{J}_k$ are characterized as minimizers of the convex but non-smooth independent optimization problems

$$\min_{\mathbf{J}_k} \left(U^k(\mathbf{J}_k) - \langle \mathbf{H}, \mathbf{J}_k \rangle + \chi_k |\mathbf{J}_k - \mathbf{J}_{p,k}| \right). \quad (2)$$

Here U^k are convex internal energy densities, $\chi_k > 0$ the pinning strengths, and $\mathbf{J}_{p,k}$ denote the values of the partial polarization from the previous load step. Fast algorithms for the solution of (2) are available [2]. The model (1)–(2) establishes an implicit relation $\mathbf{B} = \mathbf{B}(\mathbf{H}; \{\mathbf{J}_{p,k}\})$ between magnetic fields and fluxes; we write $\{\mathbf{J}_k\} = (\mathbf{J}_1, \dots, \mathbf{J}_K)$ to abbreviate the collection of the internal variables $\mathbf{J}_k$, $k = 1, \dots, K$. This *forward hysteresis operator* is very well-suited for the finite element analysis based on a reduced scalar potential [5].

II. INVERSE HYSTERESIS OPERATOR

For computations based on a vector potential, the *inverse hysteresis operator* $\mathbf{H} = \mathbf{H}(\mathbf{B}; \{\mathbf{J}_{k,p}\})$ is required instead. This relation can be evaluated numerically by inversion of the forward operator $\mathbf{B} = \mathbf{B}(\mathbf{H}; \{\mathbf{J}_{k,p}\})$ stated above, which however causes a significant computational overhead [3]. In a recent paper [4], we introduced the compact form

$$\mathbf{H} = \frac{1}{\mu_0} \left(\mathbf{B} - \sum_k \mathbf{J}_k \right) \quad (3)$$

of the inverse relation, with the partial polarization $\mathbf{J}_k$ now characterized as minimizers of a coupled problem

$$\min_{\{\mathbf{J}_k\}} F(\{\mathbf{J}_k\}) := \frac{1}{2\mu_0} \|\mathbf{B} - \sum_k \mathbf{J}_k\|^2 + \sum_k U^k(\mathbf{J}_k) + \chi_k |\mathbf{J}_k - \mathbf{J}_{p,k}|. \quad (4)$$

This non-smooth but convex minimization problem can be solved numerically by using a regularization $|x|_\epsilon := \sqrt{|x|^2 + \epsilon}$ of the Euclidean norm and applying a damped Newton method. Every step of the Newton method thus requires the solution of a coupled linear system whose dimension depends on the number K of internal variables $\mathbf{J}_k$. The complexity of evaluating the inverse hysteresis operator thus grows like $O(K^3)$, while that of the forward operator behaves like $O(K)$, since in this case the minimization problems in (2) for $\mathbf{J}_k$ can be solved separately. We now show how the evaluation of the inverse hysteresis operator can also be reduced to $O(K)$ complexity.

III. EFFICIENT REALIZATION

Let us consider the n -th step of the Newton method applied to the regularized version of (4). For convenience of notation, we introduce $g_k(\mathbf{J}_k) := U^k(\mathbf{J}_k) + \chi_k |\mathbf{J}_k - \mathbf{J}_{p,k}|_\epsilon$. The Newton updates $\{\delta \mathbf{J}_k^n\}$ are then determined by solving

$$\nabla_{\mathbf{J}_k}^2 g_k(\mathbf{J}_k^n) \delta \mathbf{J}_k^n + \frac{1}{\mu_0} \sum_{\ell=1}^K \delta \mathbf{J}_\ell^n = -\nabla_{\mathbf{J}_k} F \quad (5)$$

for all $k = 1, \dots, K$. Note that the second term introduces a coupling between all equations. To remove this coupling, we introduce the change $\delta \mathbf{J}^n := \sum_k \delta \mathbf{J}_k^n$ in the total magnetic polarization as an additional variable. This allows us to rewrite the system (5) after multiplication with μ_0 in compact form as

$$\begin{pmatrix} \mu_0 \mathbf{H}_K^n & \mathbf{I}_K \\ \mathbf{I}_K^T & -\mathbf{I} \end{pmatrix} \begin{pmatrix} \{\delta \mathbf{J}_k^n\} \\ \delta \mathbf{J}_{\text{sum}}^n \end{pmatrix} = - \begin{pmatrix} \mu_0 \nabla F(\{\mathbf{J}_k^n\}) \\ 0 \end{pmatrix}. \quad (6)$$

Here $\mathbf{H}_K^n$ denotes the block diagonal matrix with the partial Hessians $\nabla_{\mathbf{J}_k}^2 g_k^n(\mathbf{J}_k^n)$ on the diagonal and $\mathbf{I}_K$ denotes the K -fold columnwise concatenation of the identity matrix $\mathbf{I}$.

Complexity estimates. The solution of (6) can be realized efficiently via a Schur-complement technique. Since $\mathbf{H}_K^n$ is block-diagonal, we can invert it and express the $\{\delta \mathbf{J}_k^n\}$ as functions of $\delta \mathbf{J}^n$ with $O(K)$ operations. Plugging this into the second row of (6) yields a small linear system for $\delta \mathbf{J}^n$ which can be solved in $O(1)$ complexity. The updates $\{\delta \mathbf{J}_k^n\}$ can then be computed again with $O(K)$ operations. The total complexity for computing the Newton updates $\{\mathbf{J}_{k,p}\}$ thus is reduced to $O(K)$ complexity, which is the same as in for the forward hysteresis operator. In our numerical tests we in fact observe very similar computation times for evaluating the forward and inverse hysteresis operators: see Table I for details.

IV. NUMERICAL RESULTS

Next we illustrate the validity of the previous argument by some preliminary numerical results with varying number K of partial polarizations. We use the internal energy density

$$U_k(\mathbf{J}) = -\frac{Aw_k J_s}{\pi} \log \left(\cos \left(\frac{\pi}{2} \frac{|\mathbf{J}|_\epsilon}{w_k J_s} \right) \right) \quad (7)$$

with $w_k = 1/K$, $A = 50$, $J_s = 1.54$ T, and pinning forces $\chi_k = 140(k-1)/(K-1)$ for $k = 1, \dots, K$. In a first run we prescribe a series of magnetic field inputs $\mathbf{H}^i$ to the forward hysteresis operator yielding with (1) a series of magnetic fluxes $\mathbf{B}^i$ serving as input to the inverse operator. We start in a demagnetized state $\mathbf{J}_{k,p}^0 = 0$ and update in every step $\mathbf{J}_{p,k}^i = \mathbf{J}_k^{i-1}$. As expected in Figure 1 and 2 the values of

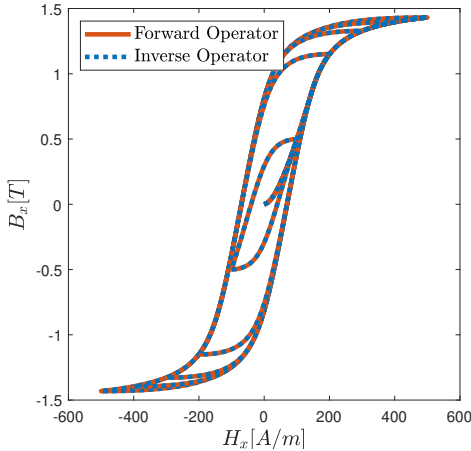


Fig. 1. Hysteresis loop of the forward and inverse operator with $K = 20$ pinning forces and an uniaxial magnetic field $\mathbf{H}^i = (H_m \sin(t^i), 0)$, where $H_m = \{100, 200, 300, 400, 500\}$.

both operators align perfectly illustrating the equality of both models. Next we consider the computation times of the forward and inverse hysteresis, where we solve for the latter the linear system in one Newton step normally (standard) and with the proposed method (6) (efficient), respectively. Note that this is only an alternative way for calculating the same solution. As postulated the computation times in Table I show that the forward operator and the inverse operator with this efficient realization can be evaluated with the same computational cost.

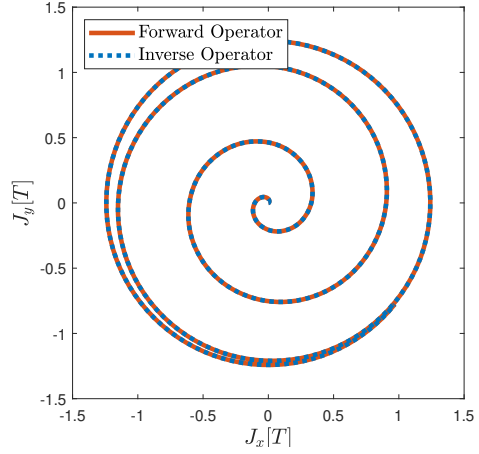


Fig. 2. Hysteresis loop of the forward and inverse operator with $K = 20$ pinning forces and a rotational magnetic field $\mathbf{H}^i = H_m^i (\sin(t^i), \cos(t^i))$, where $H_m^i = 200 \min(t^i/(6\pi), 1)$.

TABLE I
COMPUTATION TIMES

K	forward		inverse (standard)		inverse (efficient)	
	time	it	time	it	time	it
2	4ms	5.24	17ms	6.56	6ms	6.56
5	11ms	6.38	36ms	6.92	14ms	6.92
10	25ms	6.72	99ms	6.88	27ms	6.88
20	51ms	6.90	251ms	6.89	53ms	6.89
50	127ms	6.99	1186ms	6.86	131ms	6.86
80	204ms	7.00	2622ms	6.81	209ms	6.81
100	257ms	7.01	3721ms	6.79	259ms	6.79

Total computation times and average number of Newton steps for the hysteresis operators with 2000 timesteps, varying number of pinning forces K , and the magnetic field excitation $\mathbf{H}^i = (500 \sin(t^i), 0)$.

The required times for the inverse hysteresis operator without this implementation (standard) are significantly higher.

V. CONCLUSION

We have shown that the computational complexity of the inverse and forward operator in the energy-based magnetic hysteresis model have a comparable computational cost when using regularization to solve the corresponding minimization problems. As a consequence this leads to the same computation times in magnetic field simulation, which allows the efficient and accurate evaluation of iron losses in applications like electric machines and transformers. Corresponding numerical examples will be presented in the full paper.

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