

Effective Material Transformation for Laminated Iron Cores with a T, Φ - Φ Formulation and Vector Hysteresis

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In effective medium theory, a material parameter of a heterogeneous structure is replaced by an effective material, which is derived from solving a cell problem and used in a simulation with a homogenised core. With results of this simulation, the iron losses and the reactive power of machines or transformers with laminated cores are determined. For this work, a nonlinear magnetic material considering magnetic vector hysteresis is employed to simulate a single-phase transformer, providing insights into its performance.

Index Terms—Eddy currents, effective material, homogenisation method, nonlinear material, vector hysteresis

I. INTRODUCTION

SIMULATING electrical machines provides understanding of their behaviour, possibilities for optimisation, prediction of failures, and reduces development costs and time by testing virtually before physical prototyping. An accurate simulation of the field distributions in laminated iron cores requires huge finite element matrices, since a discretisation of the quasi-periodic structure is needed. Solving the resulting system of equations is challenging or even impossible.

This work addresses the modelling of nonlinear magnetic materials including vector hysteresis within the context of an eddy current problem (ECP) in electrical machines or transformers with laminated cores. The approach uses an effective material (EM) with a homogenised core based on a T, Φ - Φ -formulation [1] and approximates the appearing alternating and rotational eddy current losses (HLs) as well as the eddy current losses and the reactive power.

II. LOSSES AND REACTIVE POWER

In this paper, two different types of losses, namely eddy current losses (ECLs) and HLs, and the reactive power will be considered. The instantaneous ECLs

$$p_{eddy}(t) = \frac{1}{|\Omega|} \int_{\Omega_c} \mathbf{E} \cdot \mathbf{J} \, d\Omega, \quad (1a)$$

where Ω_c denotes an electrically conductive subdomain of Ω , $\mathbf{E}$ the electric field strength and $\mathbf{J}$ the electric current density, are determined by the purely electric quantities of the Poynting theorem. The instantaneous HLs

$$p_{hyst}(t) = \frac{1}{|\Omega|} \int_{\Omega_c} \mathbf{H} \cdot \partial_t \mathbf{B} \, d\Omega, \quad (1b)$$

where $\mathbf{H}$ denotes the magnetic field strength, ∂_t the time derivative and $\mathbf{B}$ the magnetic flux density, are determined by the purely magnetic terms. The instantaneous HLs can be separated into their alternating loss part

$$p_{alt}(t) = \frac{1}{|\Omega|} \int_{\Omega_c} |\mathbf{H}| |\partial_t \mathbf{B}| \cos(\alpha) \, d\Omega, \quad (2a)$$

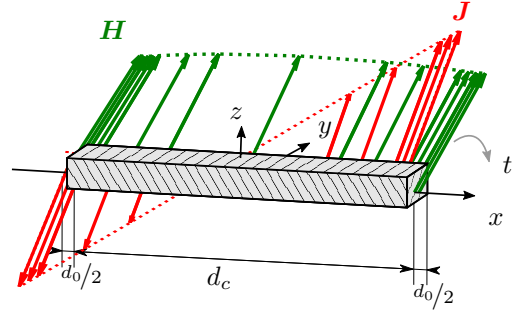


Figure 1. Single sheet for the reference cell problem of total width $d = d_0 + d_c$ with rotating magnetic field strength $\mathbf{H}$ and eddy current density $\mathbf{J}$.

where α is the lag angle between $\mathbf{H}$ and $\mathbf{B}$, and their rotational loss part

$$p_{rot}(t) = \frac{1}{|\Omega|} \int_{\Omega_c} \partial_t \theta (\mathbf{H} \times \mathbf{B}) \cdot \mathbf{e}_{rot} \, d\Omega, \quad (2b)$$

where θ is the angle of $\mathbf{B}$ and $\mathbf{e}_{rot}$ is the axis of rotation of $\mathbf{B}$, [2], [3]. Since Mayergoyz' vector hysteresis models typically do not represent rotating HLs correctly, an improvement strategy must be introduced [4].

Beside the instantaneous ECLs and the instantaneous HLs, the instantaneous fictitious reactive power (RP)

$$q(t) = \frac{1}{|\Omega|} \int_{\Omega} \mathbf{H} \cdot \mathbf{B} \, d\Omega, \quad (3)$$

is defined. Averaging the instantaneous values over one period in time

$$X = \frac{1}{T} \int_T x(t) \, dt, \quad (4)$$

where x is a placeholder for p_{eddy} , p_{hyst} , p_{alt} , p_{rot} or q , yields the corresponding losses or power per volume.

III. 3D REFERENCE PROBLEM

In the three-dimensional reference problem, each sheet in the laminated core is modelled in the mesh. The boundary value problem (BVP) of the ECP is based on Maxwell's equations

$$\nabla \times \mathbf{H} = \mathbf{J}, \quad \nabla \cdot \mathbf{B} = 0 \quad \text{and} \quad \nabla \times \mathbf{E} = -\partial_t \mathbf{B},$$

Table I
EDDY CURRENT LOSSES, HYSTERESIS LOSSES AND REACTIVE POWER

I_p	FEM			EM			Error		
	P_{eddy}	P_{hyst}	Q	$\tilde{P}_{eddy}$	$\tilde{P}_{hyst}$	$\tilde{Q}$	ε_{eddy}	ε_{hyst}	ε_Q
A	kW/m ³	kW/m ³	kVA/m ³	kW/m ³	kW/m ³	kVA/m ³	%	%	%
1	2.1	6.4	16.3	2.2	6.1	16.1	4.9	-5.3	-1.2
3	8.4	12.6	82.5	8.7	12.1	82.2	3.8	-4.2	-0.4

Table II
ALTERNATING AND ROTATIONAL HYSTERESIS LOSSES

I_p	FEM		EM	
	P_{alt}	P_{rot}	P_{alt}	P_{rot}
A	kW/m ³	kW/m ³	kW/m ³	kW/m ³
1	6.3	0.2	6.0	0.1
3	12.1	0.5	11.7	0.4

the constitutive relation $\mathbf{E} = \rho \mathbf{J}$ and the magnetic flux density $\mathbf{B} = \mathbf{\Gamma}[\mathbf{H}]$ defined by the vector Preisach model. The three dimensional reference solution is solved using the $\mathbf{T}, \Phi$ - Φ formulation [5].

IV. CELL PROBLEM

The BVP of the one-dimensional cell problem (CP), see Fig. 1, is formulated to calculate the effective permeability $\mu_{eff}(H_y, H_z)$. A single sheet of total width $d = d_c + d_0$, with the width d_c of the conducting sheet and the total width d_0 of the insulation layer, is considered which is excited by $\mathbf{H}_0 = H_{0,y}(x, t)\mathbf{e}_y + H_{0,z}(x, t)\mathbf{e}_z$ in the tangential yz -plane. The BVP of the one-dimensional CP with $\mathbf{H} = H_y(x)\mathbf{e}_y + H_z(x)\mathbf{e}_z$ reads as

Find $(H_y, H_z) \in \mathbf{V}_D := \{(H_y, H_z) \in \mathcal{U} \times \mathcal{U} : \mathbf{H} = \mathbf{H}_0(t) \text{ on } \partial\mathcal{I}\}$ such that

$$\begin{aligned} & \int_{\mathcal{I}} \Delta t \rho (\partial_x \mathbf{H} \cdot \partial_x \mathbf{H}) + \mu_{FP} \mathbf{H} \cdot \mathbf{H}' \, dx \\ &= \int_{\mathcal{I}} \widehat{\mathbf{B}} \cdot \mathbf{H}' \, dx + \int_{\mathcal{I}} (\mu_{FP} \mathbf{H}_n - \widehat{\mathbf{B}}_n) \cdot \mathbf{H}' \, dx \end{aligned} \quad (5)$$

for all $\mathbf{H}' \in \mathbf{V}_0$, where $\mathcal{I} = [-d/2, d/2]$, $\mathcal{U} \subset H^1(\mathcal{I})$ and the electric resistivity is suitable in air. The test functions are denoted by the prime symbol, values of the previous time step by the hat symbol and values of the previous nonlinear iteration by the index n .

V. HOMOGENISED CORE AND EFFECTIVE MATERIAL

In order to approximate the iron losses and the RP of the reference problem, the complex-valued equivalent static magnetic field problem (SMFP) with the nonlinear EM $\mu_{eff}(\mathbf{H})$ is solved. The weak form of the equivalent BVP reads as:

Find $\underline{\Phi} \in \mathbf{V}_D(\Omega) : \{\underline{\Phi} \in \mathcal{U} : \underline{\Phi} = 0 \text{ on } \Gamma_H\}$ such that

$$\int_{\Omega} \mu_{eff}(\mathbf{H}) \nabla \underline{\Phi} \cdot \nabla \underline{\Phi}' \, d\Omega = \int_{\Omega} \mu_{eff}(\mathbf{H}) \mathbf{H}_{BS} \cdot \nabla \underline{\Phi}' \, d\Omega \quad (6a)$$

in the conducting domain and

$$\int_{\Omega} \mu_0 \nabla \underline{\Phi} \cdot \nabla \underline{\Phi}' \, d\Omega = \int_{\Omega} \mu_0 \mathbf{H}_{BS} \cdot \nabla \underline{\Phi}' \, d\Omega \quad (6b)$$

in the non-conducting domain, for all $\underline{\Phi}' \in \mathbf{V}_0$ and $\mathcal{U} \subset H^1(\Omega)$. The known Biot-Savart field is denoted by $\mathbf{H}_{BS}$.

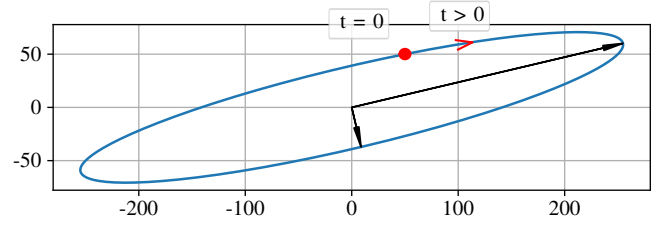


Figure 2. Corresponding time domain locus of the magnetic field strength $\mathbf{H} = (50 + 250j)\mathbf{e}_x + (50 + 50j)\mathbf{e}_y \text{ Am}^{-1}$ with annotated starting point ($t=0$), semi-minor and semi-major axes.

A. Rotational Locus

In the time-domain the magnetic field strength $\mathbf{H}(t)$ is generating a locus over time, see Fig. 2. This locus can be approximated by an ellipse based on the semi-minor and semi-major axes. A complex-valued $\underline{\mathbf{H}}$ of the homogenised core using the EM in a SMFP, corresponds to a time domain field $\tilde{\mathbf{H}}(t) = \text{Re}\{\underline{\mathbf{H}}e^{j\omega t}\} \forall t \in [0, T]$, which forms an elliptic locus as well. To find the semi-minor and semi-major axes of the locus, the maximum and the minimum of $|\underline{\mathbf{H}}|$ are the roots of $|\underline{\mathbf{H}}|^2 \sin(2\omega t + 2\varphi_H) + |\underline{\mathbf{H}}|^2 \sin(2\omega t + 2\varphi_H) = 0 \forall t \in [0, T]$ with $\varphi = \arg(\underline{\mathbf{H}})$, respectively. These values are subsequently required for the evaluation of the EM.

VI. NUMERICAL EXAMPLE

As a numerical example, the single-phase transformer described in [6] with 184 insulated sheets and the material parameters of the Preisach model in [5] are used. Two current-values $I_p \in [1, 3] \text{ A}$ were impressed into the current-driven coils. The resulting iron losses and the reactive power are summarised in Tab. I and Tab. II. In comparison, the simulation time is reduced from 45 h for the three-dimensional reference problem to about 1 min for the homogenised problem using the EM.

ACKNOWLEDGEMENT

This research was funded in whole or in part by the Austrian Science Fund (FWF) grant DOI:10.55776/P36395. For open access purposes, the author has applied a CC BY public copyright license to any author accepted manuscript version arising from this submission.

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