

Analytical Model for Sinusoidal-Coil Eddy Current Position Sensors

Jilan Huang¹, Guosi Chen², Huanran Wang¹, Ye Ma¹, Wentao Geng¹, Yukun Zhang¹, Xiaoyan Huang¹

¹College of Electrical Engineering, Zhejiang University, Hangzhou, 310027, China

²Changzhou Huaxuan Sensing Technology Co., Ltd, Changzhou, 213100, China

Among the means of sensor-based position detection, eddy current absolute position sensors have inherent advantages over magnetic encoders and resolvers in terms of size, cost, and robustness. To address the complexity of existing FEA based design methods, this paper proposed a general analytical model for sinusoidal-coil eddy current position sensors, clarifying the quantitative relationship between the output voltage and design parameters. The accuracy of the analytical model is validated through both FEA and experimental tests.

Index Terms—angular position measurement, eddy current sensor, inductive transducer, analytical models, design optimization.

I. INTRODUCTION

Absolute angular position measurement is critical in a wide range of electric machine closed-loop control systems. Magnetic sensors used in position measurement can be categorized into two types based on excitation methods. Hall effect sensors exemplified by magnetic encoders are susceptible to harsh conditions[1]. Conversely, electromagnetic induction-based resolvers exhibit better environmental robustness[2], but their larger size and weight restrict widespread applications.

Eddy current sensors (ECS) feature smaller size, low cost, and high robustness[3]. Among these ECSs, the sinusoidal-coil ECS (SECS) stands out, with the ability to support through-shaft and arc-shaped designs.

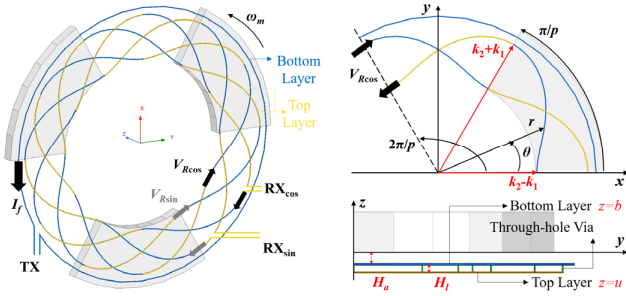
The main problem of existing design methods rely on Finite Element Analysis (FEA)[4] is the high computational demand and the difficult in explicitly expressing the relationship between parameters and performance. This paper proposes an analytical model for SECS to provide a guideline for efficient optimal design. The accuracy of the analytical model is validated through both FEA and experimental tests.

II. ANALYTICAL MODELING

Fig. 1 shows a sensor applied to a 3-pole-pair motor ($p = 3$). The excitation coil (TX) and receiving coils (RXs) are printed on the two layers of PCB. RXs comprise two structurally identical coils (RX_{sin} and RX_{cos}) with a spatial phase difference of $\pi/2p$. Each coil is distributed across two layers through vias. The conductive rotor and the PCB are separated by an airgap. The shapes of clockwise (R₊) and counterclockwise (R₋) turns of RX_{cos} are expressed in polar coordinates as

$$\begin{aligned} r_+ &= k_1 \cos p\theta + k_2 \\ r_- &= -k_1 \cos p\theta + k_2 \end{aligned} \quad (1)$$

where k_1 and k_2 are radial dimensions of RXs shown in Fig. 1.



(a) Illustration of the layered structure. (b) Geometry parameterization
Fig. 1. The 3-pole-pair model of sinusoidal-coil eddy current sensor.

Fig. 2(a) shows a geometry model to calculate the induced voltage of RX_{cos} (V_{Rcos}). The common-mode component of the flux linked with R₊ and R₋ (the eddy current magnetic flux Φ_e from rotor in area A_{co} and the excitation flux from TX) is counteracted due to the rotational symmetry and reverse series connection of R₊ and R₋, leaving only the differential-mode component (Φ_e in area A_{di}). A_{di} is enclosed by lines equally distributed at $z = u, b$, so it can be equated to an “intermediate plane” S_{eq} at $z = (u + b)/2 = h$, thus V_{Rcos} can be expressed as

$$V_{Rcos} = -N \frac{d}{dt} \int_{S_{eq}} \mathbf{B}_e \cdot d\mathbf{S}_{eq} \quad (2)$$

$\mathbf{B}_e$ is the eddy current magnetic flux density. To calculate $\mathbf{B}_e$, a simplified 2D model is introduced as shown in Fig. 2(b). In a radial cross-section of the TX-airgap-rotor system, the effect of the edge of the rotor solid is negligible for the eddy current distribution, thus the rotor can be considered as an infinite flat plate with respect to TX. Taking the Coulomb gauge of the vector magnetic potential, magnetic flux density $\mathbf{B}$ and vector magnetic potential $\mathbf{A}$ in region I satisfy diffusion equation

$$\nabla^2 \mathbf{B}^I - \mu_0 \sigma_c \frac{\partial \mathbf{B}^I}{\partial t} = 0, \quad \nabla^2 \mathbf{A}^I - \mu_0 \sigma_c \frac{\partial \mathbf{A}^I}{\partial t} = 0 \quad (3)$$

where σ_c is the conductivity of rotor material.

$\mathbf{A}$ in region II satisfy Poisson equation while $\mathbf{B}$ in region II satisfy the magnetic quasi-static equation

$$\nabla \times \mathbf{B}^{II} = \mu_0 \mathbf{J}_f, \quad \nabla^2 \mathbf{A}^{II} = -\mu_0 \mathbf{J}_f \quad (4)$$

where $\mathbf{J}_f$ is the line current density of I_f , represented by

$$\mathbf{J}_f = I_f \delta(y) \delta(z - h) \mathbf{e}_z \quad (5)$$

Solved by the Fourier transform method, the z -direction component of $\mathbf{B}^{II}$ is calculated as

$$B_z^{II} = \frac{1}{\pi} \int_0^\infty j \left(\frac{\beta k (k - \alpha)}{\alpha + k} e^{-kz} + \frac{\mu_0 I_f}{2} e^{-k|z-h|} \right) e^{iky} dk \quad (6)$$

where $\alpha = \sqrt{k^2 + j\omega_f \mu_0 \sigma_c}$, $\beta = (\mu_0 I_f / 2k) e^{-kh}$, k is the frequency variable of the Fourier transform.

In the expression of B_z^{II} , the former term is the part coming from eddy current, B_{ez}^{II} . To simplify the calculation of V_{Rcos} , an average magnetic flux density $\bar{B}_z$ is defined as

$$\bar{B}_z = \frac{\int_{S_{rotor}} B_{ez}^{II} dS_{rotor}}{S_{rotor}} = \bar{B}_{zm} \sin \omega_f t \quad (7)$$

Its amplitude $\bar{B}_{zm}$ remains constant during rotor rotation, and ω_f is the frequency of I_f . An empirical coefficient $K_B = 0.99$ is introduced to correct $\bar{B}_z$ to an equivalent-average magnetic flux density in the linked plane S_{cover} , which is defined by

$$\hat{B}_z = K_B \cdot \bar{B}_z = K_B \bar{B}_{zm} \sin \omega_f t \quad (8)$$

Consider that $\hat{B}_z$ is uniformly distributed on S_{cover} . Based on $\hat{B}_z$, the equation for V_{Rcos} can be simplified as

$$V_{Rcos} = -pN \frac{d}{dt} (\hat{B}_z(t) \cdot S_{cover}(t)) \\ = -pN \frac{d}{dt} \hat{B}_z(t) \cdot \left(\int_{\frac{3\pi}{2p}}^{\frac{\pi}{2p}} -2k_1 \cos\left(\frac{p}{R_i}x\right) dx - \int_{\frac{\omega_m R_i + \frac{\pi}{p} R_i}{2p}}^{\frac{\omega_m R_i + \frac{\pi}{p} R_i}{2p}} -2k_1 \cos\left(\frac{p}{R_i}x\right) dx \right) \quad (9)$$

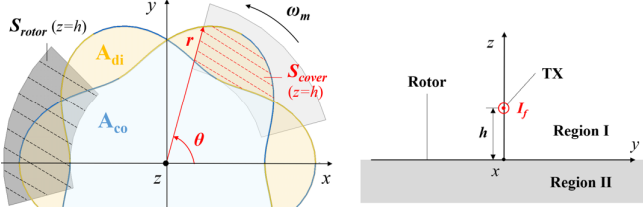
where $R_i = k_2 - k_1$, $x = \theta R_i$.

According to (8) and (9), V_{Rcos} is calculated as

$$V_{Rcos} = 4Nk_1(k_2 - k_1)\omega_f K_B \bar{B}_{zm} \cdot \sin p\omega_m t \cdot \cos \omega_f t \quad (10)$$

Similarly, the induced voltage of R_{sin} can be expressed as

$$V_{Rsin} = 4Nk_1(k_2 - k_1)\omega_f K_B \bar{B}_{zm} \cdot \cos p\omega_m t \cdot \cos \omega_f t \quad (11)$$



(a) Geometry model to calculate V_{Rcos} (b) 2D electromagnetic model
Fig. 2. Illustration of the two-part physical structure of the analytical model.

III. FEA VERIFICATION

The FEM simulation parameters are presented in Table I.

TABLE I
FEA SIMULATION PARAMETERS

Parameters	Value	Unit
k_1/k_2	2.8 / 31.8	mm
Airgap thickness H_a / PCB thickness H_l	0.8 / 0.2	mm
Rotor speed	16000	rpm
Excitation current frequency ω_f / RMS $I_{f rms}$	3/20	MHz/mA
Turns of R_{Xsin}/R_{Xcos}	2	-
Conductivity of rotor material	3.8×10^7	S/m

A. Finite Element Simulation Results

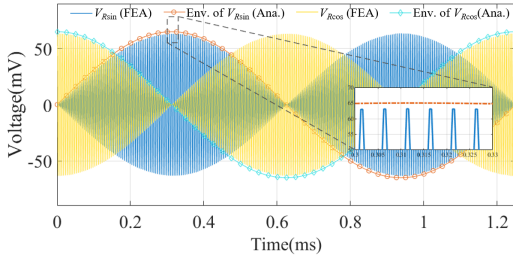


Fig. 3. Comparison of signals obtained from FEA and analytical model.

Fig. 3 shows V_{Rsin}, V_{Rcos} from FEA and their envelopes from analytical model (Ana.), validating the effectiveness of the analytical model. Due to the edge effect of the rotor, the eddy current density is not uniformly distributed, thus equating B_e to the uniformly distributed $\hat{B}_z$ introduces an error.

B. Validation of Parameter Analysis Using Analytical Model

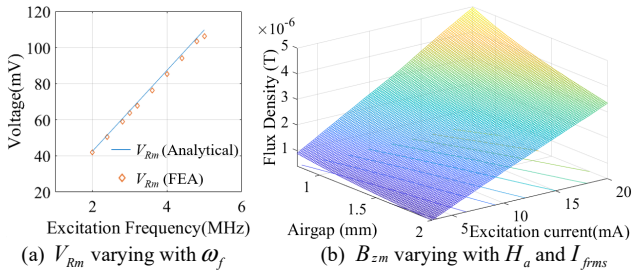


Fig. 4. Parameter and response surface analysis of several parameters

Taking the relationship between the amplitude of V_R (V_{Rm}) and ω_f (Fig. 4(a)), as well as the response surface of with respect to H_a and $I_{f rms}$ (Fig. 4(b)) as examples, the results calculated analytically are consistent with those from FEA. Further parameter analysis will be presented in the full paper.

IV. EXPERIMENTAL VERIFICATION

A high-speed driving experimental platform integrated with the sensor was set up, as shown in Fig. 5, with parameters listed in Table I. V_{Rsin} at 16,000 rpm is shown in Fig. 6, which aligns with analytical calculation results.

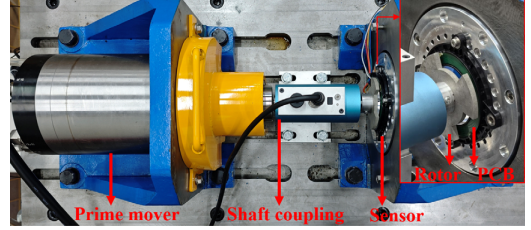


Fig. 5. Experimental platform.

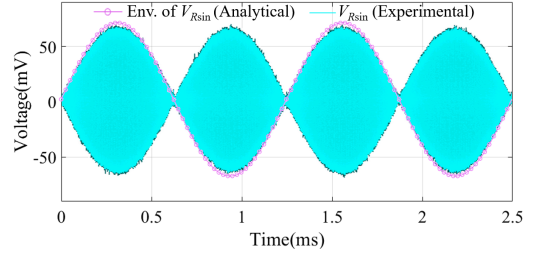


Fig. 6. Comparison of signals obtained from experiment and analytical model.

V. CONCLUSION

This paper proposed an analytical model for the SECS, establishing a clear quantitative relationship between its output voltage and structural as well as electromagnetic parameters. Compared to FEA, it provides a simpler and faster means to evaluate the impact of various parameters on performance. The accuracy of the analytical model is verified through FEA and experimental tests, with errors between analytical calculations and FEA as well as experimental results not exceeding 1%. The model offers a guideline for efficient optimization design.

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