

Study of an Angle-Dependent Parameter J-A Model Based on Magnetic Anisotropy

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This digest introduces an enhanced Jiles Atherton(J-A) model with angle-dependent parameters to achieve high-precision magnetization computations for permalloy alloys under conditions of anisotropic magnetic permeability. The conventional J-A model typically employs a uniaxial approach, fitting magnetization curves or hysteresis loops only along the easy magnetization axis. However, permalloy exhibits pronounced magnetic anisotropy, and in practical applications, magnetization or demagnetization may occur in any arbitrary direction. Therefore, the fitting of single axis leads to significant errors in magnetization predictions. To resolve this issue, the hysteresis and magnetization curves of permalloy sheets at various directions were measured and a nonlinear mathematical model for angles and JA parameters is established. The angle-dependent parameter J-A(ADP·J-A) model perform the magnetic field calculation using the magnetization parameters corresponding to the specific direction.

Index Terms—angel-dependent parameter J-A model, anisotropic magnetic properties, magnetic shielding material, magnetization state calculation.

I. INTRODUCTION

A nearly zero magnetic field environment[1] is a crucial prerequisite for atomic gyroscopes, magnetometers, medical and biomedical research. Magnetically shielded rooms (MSRs) are constructed using high-conductivity materials such as permalloy. Precise calculation of the magnetization state of permalloy is essential for MSRs under diverse operational conditions, for which the J-A hysteresis model is a crucial tool. It is based on five parameters M_s , a , α , c and k , which represent the magnetic properties of the material. Shi et al. developed a calculation model using the dynamic J-A method, based on the hysteresis loops of permalloy sheets under demagnetization excitation[2]. Yang et al. used J-A model to verify that magnetic vibration can enhance the shielding effectiveness of magnetic shielding devices[3]. However, the above studies have ignored the influence of anisotropic magnetic properties. The vector J-A model introduces the anisotropy of material by extending each parameter into three components corresponding to the three-dimensional coordinate axes[4].

Therefore, an ADP·J-A model is proposed, which adjusts parameters based on the magnetization direction. Magnetization curves at 0°, 30°, 45°, 60°, and 90° relative to the hard (0°) and easy (90°) magnetization axes are measured for a permalloy sheet. The measured data is used to mathematically fit the relationship between angle and anisotropic magnetic properties, forming a continuous, nonlinear model. The ADP·J-A model, with direction-dependent parameter corrections, significantly improves the accuracy of magnetization states under arbitrary excitations.

II. DATA TESTING AND ADP·J-A MODELING

This study utilizes a self-developed two-dimensional SST measurement device to measure the B-H curves of permalloy

plates at angles of 0°, 30°, 45°, 60°, and 90°. The schematic diagram of the device and the results are shown in Fig. 1. Each curve is composed of the vertices of 28 symmetric hysteresis loops.

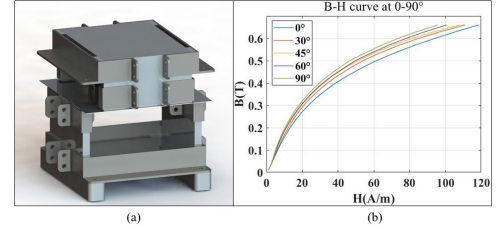


Fig. 1. 2D-SST structure diagram and test data. (a) 2D-SST structure diagram. (b) B-H curve at 0-90°

The anhysteretic magnetization of J-A model can be expressed by Langevin function:

$$M_{an} = M_s \left(\coth\left(\frac{H_e}{a}\right) - \left(\frac{a}{H_e}\right) \right) \quad (1)$$

Where M_{an} indicates the anhysteretic magnetization, M_s is the saturated magnetization, a is the domain wall density, and H_e represents the effective field and can be expressed as:

$$H_e = H + \alpha M \quad (2)$$

Where α is the mean field parameter representing the inter-domain coupling. The relationship between reversible and irreversible magnetization can be expressed as:

$$M_{rev} = c(M_{an} + M_{irr}) \quad (3)$$

where c is reversible magnetization coefficient. The energy balance equation can be expressed as:

$$\mu_0 \int M dH_e = \mu_0 \int M_{an} dH_e - (1 - c)\mu_0 k \delta \int \frac{dM_{irr}}{dH_e} dH_e - \int \frac{\mu_0^2 d^2}{2\rho\beta} \left(\frac{dM}{dt}\right)^2 dt \quad (4)$$

where the $\delta=1$ when $dH_e \geq 0$, the $\delta = -1$ when $dH_e < 0$.

The basic principle of the ADP·J-A model is illustrated in "Angle-dependent Parameters" in Fig. 2. After the excitation was determined, the direction of the excitation, denoted as θ , is

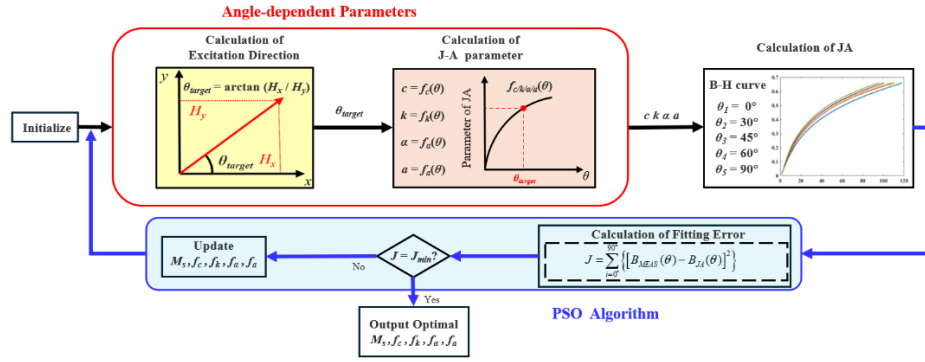
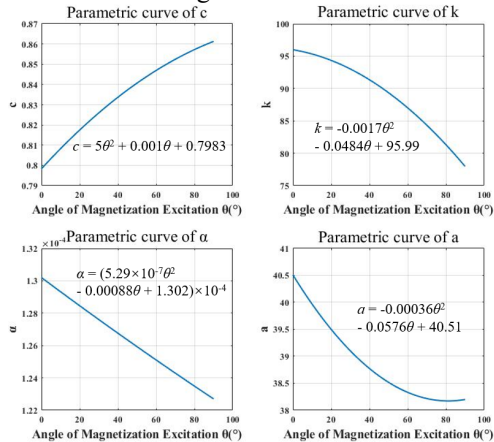


Fig. 2. ADP·JA Fitting Flowchart.

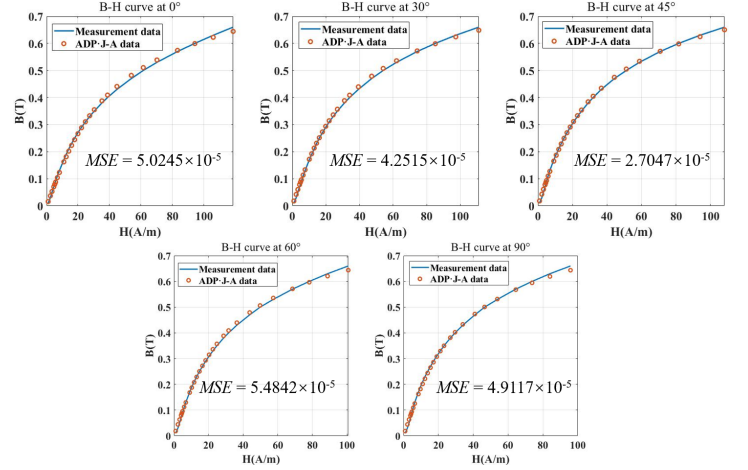
first calculated and then passed to the parameter calculation section. Subsequently, the θ is substituted into the respective angle-dependent functions for c , k , α and a to calculate the corresponding parameters for the JA model under the current excitation. Due to the involvement of five parameters corresponding to angular variation functions, the optimization problem features a high number of parameters and a nonlinear relationship between angle and magnetic properties. Therefore, the Particle Swarm Optimization (PSO) algorithm is employed for fitting. The population size is set to 100, and the maximum number of iterations is 300.

In this paper, the fitting process of the ADP·J-A model is illustrated in Fig. 2. The process is described as follows: Step 1: Initialization. Excitation is created and the initial state of the calculation functions for parameters are determined; Step 2: Angle-dependent parameter calculation. The "Angle-dependent Parameters" section is proceeded to calculate the parameters; Step 3: B-H curve calculation. The calculated parameters are used to compute the B-H curve based on the J-A model. In this study, basic magnetization curves at 0° , 30° , 45° , 60° and 90° are used for fitting. (The basic magnetization curve data is obtained from a two-dimensional single sheet tester.) Step 4: Error calculation and summation. Comparing the computed B-H curve with the measured B-H curve to calculate the error. Step 5: Optimization with PSO. Applying the Particle Swarm Optimization (PSO) algorithm to determine the optimal function.

The forms of fitting functions are all quadratic functions, and the results are shown in Fig. 3.



The comparison between the measured B-H curves and those predicted by the ADP·J-A model, along with the associated errors, at 0° , 30° , 45° , 60° , and 90° is shown in Fig. 4.



For this model, the fitting errors at 0° , 30° , 45° , 60° , and 90° are all below 1×10^{-4} , indicating excellent fitting accuracy.

III. CONCLUSION

This paper presents an angle-dependent parameter J-A model based on magnetic anisotropy (ADP · J-A). The study employs 2D-SST device to measure the B-H curves of permalloy sheets at 0° - 90° . A nonlinear model is established to describe the relationship between magnetic properties and angles, improving the calculation and simulation accuracy of the J-A model under anisotropic conditions.

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