

Modal Analysis of Leakage Magnetic Field from Line Currents Within Rectangular Conducting Duct With a Slot

Hyun Ho Park, *Senior Member, IEEE*

School of Electrical and Electronic Engineering, The University of Suwon
Hwaseong, 18323 KOREA, hhpark@suwon.ac.kr

This paper presents an analytical solution for a two-dimensional structure consisting of long current-carrying lines within a rectangular and slotted duct using the mode-matching method. The solution facilitates efficient and accurate calculation of leakage magnetic fields by accounting for enclosure and slot dimensions, as well as the number, magnitude, phase, and position of the current lines within the duct. Compared to numerical methods, the proposed series solution offers faster computation while maintaining accuracy and convergence, providing a robust tool for electromagnetic analysis.

Index Terms—Leakage magnetic field, long line current, Green’s function, magnetic vector potential, mode-matching method, rectangular enclosure, slot

I. INTRODUCTION

THE rapid advancement of the artificial intelligence (AI) industry has driven the proliferation of data centers in metropolitan areas. Data centers require substantial electrical power, necessitating the use of high-voltage and high-current transmission lines to supply the power. To accommodate these data centers within urban environments, such transmission lines must often be buried underground. However, deeply embedding these high current cables incurs significant time and financial costs, leading to a practical solution of installing them at depths of less than 1 meter. This shallow installation poses challenges, as the magnetic fields generated by high-voltage and high-current transmission lines can adversely affect pedestrians and nearby electronic devices. Consequently, shielding structures designed to attenuate these magnetic fields are essential.

Various methods have been developed to analyze and mitigate magnetic fields from underground power cables. A theoretical 2D model using variable separation for multilayered conductive shields was proposed in [1], providing explicit expressions for magnetic fields and power loss. In [2], a hybrid FEM-BEM approach analyzed optimal arrangements of conductive and ferromagnetic materials for multilayer plates and reversed U-shaped shields. Similarly, [3] adopted a hybrid FEM/semi-analytical method for open boundary problems in underground cable systems. An approximate method for calculating magnetic fields near finite-width-thick plates was introduced in [4]. Additionally, [5] explored magnetic field mitigation using shielding plates, considering shielding effectiveness, series impedance, and power loss contributions.

This paper derives an analytical solution for a two-dimensional (2D) structure comprising long current lines within a rectangular enclosure-shaped duct with slots, based on the mode-matching method [6], [7]. The solution enables efficient calculation of the leakage magnetic field by consid-

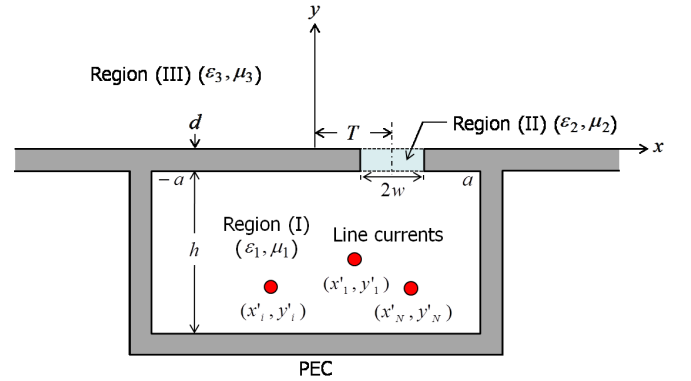


Fig. 1: Two-dimensional (2D) geometry of long line currents within slotted rectangular conducting duct.

ering the size of the rectangular duct and slots, as well as the number, magnitude, phase, and position of the long current lines within the duct. The proposed series solution offers the advantage of faster computation times compared to numerical methods while ensuring accurate and convergent results.

II. FORMULATION

The magnetic fields from low-frequency power lines are modeled using a quasi-static vector magnetic potential formulation. The 2D structure in Fig. 1 is divided into three regions: the 2D rectangular duct and slot regions use Fourier series for the vector potential, while the semi-infinite plane above the slotted conducting plate employs Fourier integral forms. Multiple line currents are represented as

$$I^{(i)}(x, y) = I_z^{(i)} \delta(x - x'_i) \delta(y - y'_i), \quad (1)$$

where $I_z^{(i)} = |I_z^{(i)}| e^{i\theta^{(i)}}$, $|I_z^{(i)}|$ and $\theta^{(i)}$ are the magnitude and phase of the i -th line current in the rectangular duct. Since the long line currents inside the conducting duct act as sources of the vector potential, the vector potential from the line current sources in the rectangular duct is expressed using the Green’s

function of a 2D rectangular cavity. The vector potential from multiple line currents is given by

$$\begin{aligned} A_z^s(x, y) &= \mu_1 \oint_{S_i} I(x'_i, y'_i) G(x, y|x', y') dx' dy', \\ &= \mu_1 \sum_{i=1}^N I_z^{(i)} G(x, y|x', y'), \end{aligned} \quad (2)$$

where

$$\begin{aligned} G(x, y|x', y') &= \sum_{n=1}^{\infty} \frac{2 \sin[a_n(x+a)] \sin[a_n(x'+a)]}{n\pi \sinh(a_n h)} \\ &\times \begin{cases} \sinh[a_n(y+d)] \sinh[a_n(y'+d+h)], & y' < y < -d \\ \sinh[a_n(y'+d)] \sinh[a_n(y+d+h)], & -d-h < y < y' \end{cases} \end{aligned} \quad (3)$$

and $a_n = n\pi/(2a)$. In each region, the vector potentials are

$$A_z^I(x, y) = \mu_1 \sum_{n=1}^{\infty} e_n \sin[a_n(x+a)] \sinh[a_n(y+d+h)], \quad (4)$$

$$\begin{aligned} A_z^{II}(x, y) &= \mu_2 \sum_{m=1}^{\infty} \sin[w_m(x-T+w)] \\ &\times [b_m \cosh[w_m(y+d)] + c_m \sinh[w_m(y+d)]], \end{aligned} \quad (5)$$

$$A_z^{III}(x, y) = \frac{\mu_3}{2\pi} \int_{-\infty}^{\infty} \tilde{A}_z^{III}(\zeta) e^{-i\zeta x - |\zeta|y} d\zeta, \quad (6)$$

where $w_m = m\pi/(2w)$ and $\tilde{A}_z^{III}(\zeta)$ is the inverse Fourier transform of $A_z^{III}(x, 0)$ given by $\tilde{A}_z^{III}(\zeta) = \int_{-\infty}^{\infty} A_z^{III}(x, 0) e^{i\zeta x} dx$. The medium of each region can be magnetic material representing the permeabilities, μ_1 , μ_2 , and μ_3 .

Using $\mathbf{B} = \nabla \times \mathbf{A}$, the tangential and normal components of magnetic flux densities B_x and B_y can be formulated. To determine unknown modal coefficients b_m and c_m , it is necessary to enforce the boundary conditions which are the continuities of the normal magnetic flux density and the tangential magnetic field intensity at $y = 0$ and $y = -d$. A simultaneous equation at $y = 0$ is

$$\begin{aligned} &[b_p \sinh(w_p d) + c_p \cosh(w_p d)] w \delta_{mp} \\ &= \frac{\mu_2}{2\pi \mu_3} \sum_{m=1}^{\infty} [b_m \cosh(w_m d) + c_m \sinh(w_m d)] w_m I_{mp}, \end{aligned} \quad (7)$$

where $p = 1, 2, 3, \dots$, δ_{mp} is the Kronecker delta, and

$$\begin{aligned} I_{mp} &= \int_{-\infty}^{\infty} |\zeta| F_m(\zeta) F_p(-\zeta) d\zeta, \\ &= \begin{cases} 2\pi w/w_p - 2[2 - (p\pi) \text{si}(p\pi)]/w_p^2 & \text{for } m = p \\ 4[\ln(w_p/w_m) + \text{ci}(m\pi) - \text{ci}(p\pi)]/(w_m^2 - w_p^2) & \text{for } m+p = \text{even} \\ 0 & \text{for } m+p = \text{odd} \end{cases} \end{aligned} \quad (8)$$

Another simultaneous equation at $y = -d$ is

$$s_p + \frac{\mu_2 a}{\mu_1} \sum_{m=1}^{\infty} b_m w_m \left\{ \sum_{n=1}^{\infty} \frac{J_{mn} \bar{J}_{pn}}{\tanh(a_n h)} \right\} = w_p c_p w \delta_{mp}. \quad (9)$$

where

$$s_p = \sum_{i=1}^N \sum_{n=1}^{\infty} \frac{I_z^{(i)} \sin[a_n(x'_i + a)]}{a \sinh(a_n h)} \sinh[a_n(y'_i + d + h)] \bar{J}_{pn} \quad (10)$$

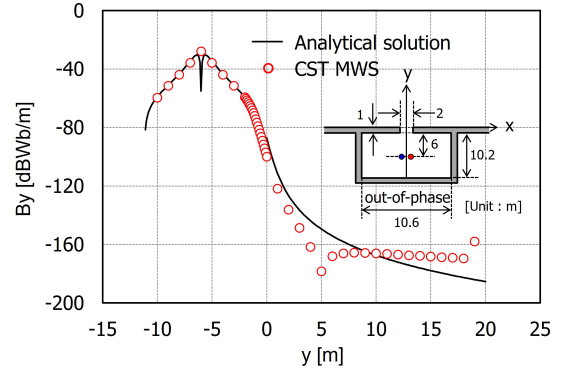


Fig. 2: Magnetic flux density (B_y) by two line currents with a 180° phase difference (out-of-phase).

and

$$J_{mn} = \begin{cases} [(-1)^m \sin[a_n(a+T+w)] - \sin[a_n(a+T-w)]] \\ \times a_n / (a_n^2 - w_m^2) \\ w \cos[a_n(a+T-w)] + [(-1)^m \sin[a_n(a+T+w)] \\ - \sin[a_n(a+T-w)]] / (4a_n) \quad (\text{if } a_n = w_m) \end{cases}, \quad (11)$$

$$\bar{J}_{pn} = \begin{cases} [(-1)^p \sin[a_n(a+T+w)] - \sin[a_n(a+T-w)]] \\ \times w_p / (a_n^2 - w_p^2) \\ w \cos[a_n(a+T-w)] - [(-1)^p \sin[a_n(a+T+w)] \\ - \sin[a_n(a+T-w)]] / (4a_n) \quad (\text{if } a_n = w_p) \end{cases}. \quad (12)$$

By matrix inversion, (7) and (9) can be solved. Using the modal coefficients b_m and c_m , the magnetic fields in all regions can be determined. Fig. 2 shows the calculation of B_y along the y -axis for two line currents with a 180° phase difference, symmetrically placed along the y -axis inside a rectangular conducting duct. The dimensions of the analysis structure are shown in the subfigure of Fig. 2, with air as the assumed medium in all regions. It is observed that the leakage magnetic flux decreases as it moves away from the duct. Numerical results obtained from CST Microwave Studio (MWS) [8] align well with the analytical solution.

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