

Kirchhoff-consistent warped Gaussian processes for electrical circuitry

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Electromagnetic (EM) design of electric vehicles (EVs) is of rising significance, since any limit violation of the EM interference (EMI) prohibits the EVs' road approval. The design for electromagnetic compatibility (EMC) of conducted emission (CE) can be stated as a multi-parametric optimization problem of Kirchhoff's Current Law (KCL), since CE tests rely on measured voltages instead of fields. Gaussian Process ($\mathcal{GP}$) Regression is known to solve these problems. The high complexity of EV traction inverter models forces to use a minimal number of training data, which could be achieved by incorporating physical constraints. We present a $\mathcal{GP}$ model that obeys KCL, paving the way to design machine learning tailored to solve circuit-level EM problems.

Index Terms—Electromagnetic compatibility, Gaussian Process, Kernel machines, Statistical learning

I. INTRODUCTION

INCORPORATING physical constraints into ML models enhances their interpretability and alignment with real-world systems. This work introduces a novel $\mathcal{GP}$ Regression framework that inherently obeys KCL, ensuring physically consistent predictions for electrical circuit currents. We formulate a $\mathcal{GP}$ prior for circuit currents, embedding the node equations through a kernel design derived from the null space of the node matrix. To build a surrogate, we extend the model with a warped $\mathcal{GP}$ and an extension to handle wideband frequency data. The proposed approach is demonstrated on an EMI filter example, validating its ability to ensure the prior distribution fully satisfies the physical constraints while providing a surrogate model capable of capturing circuit-level EM behavior across a wide frequency domain.

II. MATHEMATICAL STRUCTURE

A. Gaussian Process circuit

Let us consider a circuit with branch currents i_j (for $j = 1, 2, \dots, J$), which are governed by the node matrix $\mathbf{Q}$. The node matrix satisfies KCL,

$$\sum_{j=1}^J Q_{mj} i_j = 0. \quad (1)$$

We define a Gaussian Process prior I_j for each of the currents i_j separately:

$$I_j(\mathbf{x}) \sim \mathcal{GP}(0, k(\mathbf{x}, \mathbf{x}'; \boldsymbol{\theta}_j)) \quad (2)$$

with zero mean and kernel function $k(\mathbf{x}, \mathbf{x}'; \boldsymbol{\theta}_j)$, corresponding to the j -th component of the J currents. The inputs $\mathbf{x}$ for these kernels are the design parameters of the circuit. All kernels get the same inputs $\mathbf{x}$, but they are parameterized by different hyperparameter sets $\boldsymbol{\theta}_j$. At this point we have J marginally independent $\mathcal{GP}$ -priors describing the currents of the circuit.

B. Construction of kernel

Jidling et al. [1] proposed a method to incorporate linear constraints into Gaussian Process models. Using this approach, we aim to adapt the model in (2) to enforce the constraint specified in (1). Following the steps of [1], we establish a relationship between the Gaussian Process $\mathbf{I}(\mathbf{x})$, and an auxiliary vector-valued process $\mathbf{g}(\mathbf{x}) \sim \mathcal{GP}(\mathbf{0}, \mathbf{k}_g(\mathbf{x}, \mathbf{x}'))$, through an a priori unknown linear transformation $\mathcal{G}$. We express $\mathbf{I}(\mathbf{x})$ in terms of $\mathbf{g}$ using the transformation $\mathcal{G}$:

$$\mathbf{I} = \mathcal{G}\mathbf{g}. \quad (3)$$

With this construction, constraints can be directly introduced through the operator $\mathcal{G}$ into $\mathbf{I}(\mathbf{x})$. Since $\mathbf{g}$ is defined as a $\mathcal{GP}$ with zero mean and kernel $\mathbf{k}_g$, the $\mathcal{GP}$ -prior $\mathbf{I}(\mathbf{x})$ can be expressed as:

$$\mathbf{I}(\mathbf{x}) \sim \mathcal{GP}(\mathbf{0}, \mathcal{G}_x \mathbf{k}_g(\mathbf{x}, \mathbf{x}') \mathcal{G}_{x'}^T). \quad (4)$$

By exploiting the fact that the operator $\mathcal{G}$ will in our case be a matrix $\mathbf{G}$, we can write this transformation as:

$$\hat{\mathbf{k}}(\mathbf{x}, \mathbf{x}') = \sum_{j=1}^J \sum_{m=1}^M G_{ij} k_g(\mathbf{x}, \mathbf{x}')_{jm} G_{mn}. \quad (5)$$

To calculate the matrix $\mathbf{G}$ we look at (1) and (3) and insert them into each other:

$$\sum_{j=1}^J Q_{mj} I_j = \sum_{j=1}^J \sum_{i=1}^I Q_{mj} G_{ji} g_i = 0. \quad (6)$$

In this new equation a part can be identified that if

$$\sum_{j=1}^J Q_{mj} G_{ji} = 0 \quad (7)$$

holds, the constraint is satisfied independent of the choice of $\mathbf{g}$. This is the case if the matrix forms a basis for the null space of the matrix $\mathbf{Q}$, defined as:

$$\text{null}(\mathbf{Q}) = \text{span}\{\mathbf{v} : \sum_{j=1}^J Q_{mj} v_j = 0\}. \quad (8)$$

The matrix $\mathbf{G}$ is constructed out of the null-space of $\mathbf{Q}$ by horizontally stacking together the vectors $\mathbf{v}$ found in the span of the null-space:

$$\mathbf{G} = [\mathbf{v}^{(1)}, \mathbf{v}^{(2)}, \dots, \mathbf{v}^{(n)}] \quad (9)$$

By this construction (6) will always be fulfilled, regardless of the choice of $\mathbf{g}$.

C. Warping and Wideband Extension

To model circuit currents, data is often transformed into the dB scale, which disrupts the linear relationship (1) at the nodes. Snelson et al. [2] proposed the *warped Gaussian Process*, where the probability distribution is transformed by a symmetric, monotonic function t . Rios et al. [3] write the transformed probability density $p(\mathbf{y})$ as:

$$p(\mathbf{y}) = \prod_{i=1}^N \frac{\partial t(y_i)}{\partial y} \mathcal{N}(t(\mathbf{y}) | \mathbf{0}, \Sigma) \quad (10)$$

with $\Sigma = \hat{\mathbf{k}}(x, x')$ as the covariance matrix and N as the number of data points. We adopt this framework to apply nonlinear transformations while preserving the currents' linear relationships. To model over a wide frequency range, an extension to vector-valued predictions is necessary. Rezende et al. [4] propose a method to extend scalar GPs to vector-valued functions. Using this, we extend our $\mathcal{GP}$ prior $\mathbf{I}(\mathbf{x}) : \mathbb{R}^d \rightarrow \mathbb{R}^{J+p}$, where J is the number of currents and p the frequency positions, to predict across the entire frequency range.

III. $\mathcal{GP}$ REGRESSION MODEL OF A FILTER CIRCUIT

The specific circuit used for this example is shown in Fig. 1. The circuit has 5 currents, $i_1 - i_5$, and is defined by the

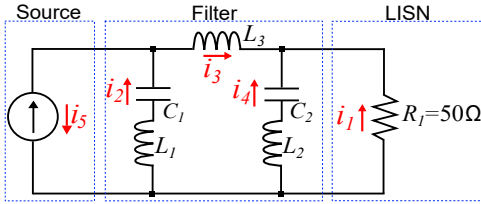


Fig. 1. Example circuit with indicated currents $i_1 - i_5$ and design parameters $\mathbf{x} = [L_1, L_2, L_3, C_1, C_2]$

node matrix $\mathbf{Q}$:

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 & -1 & -1 & 0 \\ 0 & 1 & -1 & 0 & -1 \end{bmatrix} \quad (11)$$

such that it fulfills (1). As defined before, we assign a $\mathcal{GP}$ prior to all 5 currents, with the design parameters $\mathbf{x} = [L_1, L_2, L_3, C_1, C_2]$. We choose $\mathbf{g}(\mathbf{x})$ as a 3 dimensional vector valued $\mathcal{GP}$ with a diagonal kernel function $\mathbf{k}_g = \text{diag}(k_{g1}, k_{g2}, k_{g3})$. Following the steps explained above, the matrix $\mathbf{G}$ and the new kernel can be calculated. With equation (5) the final kernel function $\hat{\mathbf{k}}$ is computed as

$$\hat{\mathbf{k}} = \begin{bmatrix} k_{g1} + k_{g2} + k_{g3} & k_{g1} & k_{g2} + k_{g3} & k_{g2} & -k_{g3} \\ k_{g1} & k_{g1} & 0 & 0 & 0 \\ k_{g2} + k_{g3} & 0 & k_{g2} + k_{g3} & k_{g2} & -k_{g3} \\ k_{g2} & 0 & k_{g2} & k_{g2} & 0 \\ -k_{g3} & 0 & -k_{g3} & 0 & k_{g3} \end{bmatrix}. \quad (12)$$

The Gaussian Process $\mathbf{I}(\mathbf{x})$ from equation (4), with this new kernel function, now inherently fulfills the node equations of the circuit defined with the matrix $\mathbf{Q}$. To show this, we can draw random samples from the $\mathcal{GP}$ -prior $\mathbf{I}(\mathbf{x})$ and check if the constraint (1) is fulfilled. In Fig. 2 random samples from the prior are shown. Up to a small noise, the samples fulfill the node constraint. With the wideband extension, we can train the

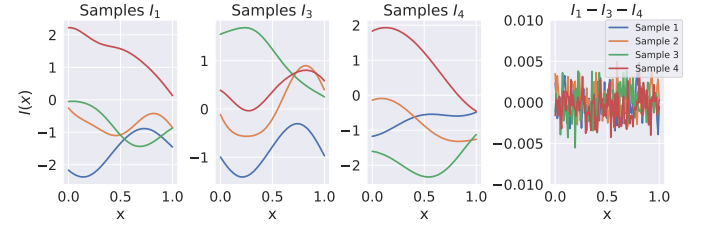


Fig. 2. Random samples of currents i_1, i_3, i_4 drawn from the $\mathcal{GP}$ -prior $\mathbf{I}(\mathbf{x})$ evaluated for a one dimensional input vector $x \in [0, 1]$.

model and make predictions for new circuit configurations over a wide frequency range. In Fig. 3 an example of a prediction of the final model is shown. Due to the numerically complex nature of the warping and the wide-band extension, further work is required to improve prediction accuracy.

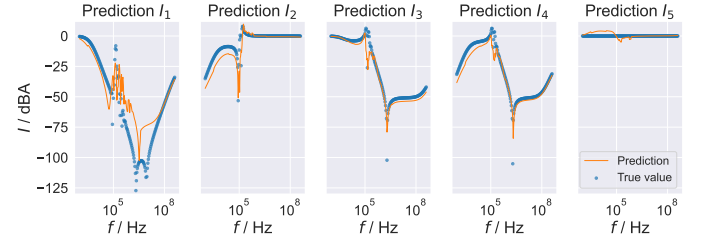


Fig. 3. Predictions of test samples using the KCL model. The model was trained using 200 circuit configurations and the simulated current data across the frequency range of 1 kHz - 400 MHz.

IV. CONCLUSION

We have developed a Gaussian Process model for a circuit, where the prior probability density inherently satisfies KCL. We verify that the prior samples satisfied the constraint and present an example of a wideband prediction model. While further work is needed to enhance the model's prediction accuracy, the physical consistency of the prior provides a strong foundation for future improvements. With these advances, the proposed framework has the potential to enable surrogate models tailored to specific circuit topologies and pave the way for physically constrained EM models.

REFERENCES

- [1] C. Jidling, N. Wahlström, A. Wills, and T. B. Schön. Linearly constrained gaussian processes. *NeurIPS* 2017, 2017.
- [2] E. Snelson, Z. Ghahramani, and C. Rasmussen. Warped gaussian processes. In S. Thrun, L. Saul, and B. Schölkopf, editors, *Advances in Neural Information Processing Systems*, volume 16. MIT Press, 2003.
- [3] G. Rios and F. Tobar. Compositionally-warped gaussian processes. *Neural Networks*, 118:235–246, 2019.
- [4] R. S. Rezende, J. Hansen, A. Piwonski, and R. Schuhmann. Wideband kriging for multiobjective optimization of a high-voltage EMI filter. *IEEE Transactions on Electromagnetic Compatibility*, 66(4):1116–1124, 2024.