

Multiscale Semi-analytical Homogenization for Analysis of Litz Wire

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This paper proposes the multiscale homogenization method for the analysis of Litz wire. By sequentially homogenizing Litz wire at the strand, bundle and winding levels, the coil region can be approximated as a uniform material with the complex relative permeability. The accuracy of the method is validated by comparing the computed relative permeability, stored energy, and Joule loss with finite element model.

Index Terms— Eddy currents, Litz wire, multiscale extended Ollendorff.

I. INTRODUCTION

It has been shown that the semi-analytical homogenization method based on the extended Ollendorff formula and complex permeability expressed in a closed form can effectively simplify models of coils in electrical equipment, enhancing numerical computation efficiency without deterioration of accuracy [1]. The advantage of the semi-analytical homogenization lies in its ability to analyze the magnetic energy and eddy current losses in the frequency domain using magnetostatic field analysis without finite discretization considering the skin depth.

In practical applications, many devices have coils that are composed of several turns of Litz wire. Each turn consists of multiple conductor strands twisted together, isolation builds, and the outer insulating material, as shown in Fig. 1. In other words, coils can be divided into the strand, bundle, and winding levels with different scales. Each level corresponds to its own lumped circuit model shown in Fig. 2.

However, the conventional semi-analytical homogenization cannot directly deal with aforementioned coils at the winding level, which requires multi-scale processing. Multiscale homogenization is not a new concept; however, there has been no report on the semi-analytical homogenization method. In addition, if we can establish a uniformly equivalent region using the proposed method for the coils in magnetostatic fields, it will enable subsequent study on coil shape optimization taking into account the eddy current effect.

In this paper, after formulating the proposed method, the stored magnetic energy and Joule loss due to eddy currents computed by the proposed model with coarse mesh and conventional model with sufficiently fine finite elements are compared for the validation.

II. METHODOLOGY AND PROCESS

We consider the Litz wire with multiscale structure shown in Fig. 1. Let us assume the input current is 30 A, with the frequency range f_{re} between 50 kHz and 200 kHz. The skin depth δ is then between 0.146 mm and 0.291 mm. In practical design, the diameter of the stranded wire is selected so that it does not exceed twice the skin depth to reduce the eddy current loss. Thus, at the strand level, AWG 32 is selected, with a single isolation build of 1/20 [2]. Then moving on to the bundle level,

to ensure safety at the present current level, the cross-sectional area of Litz wire to form a single turn must not be less than 2.5 mm² [3]. Therefore, at least 77 strands of wire are required, and we choose 80 strands in this case. In addition, each turn of the coil is covered with a 0.2 mm ETFE insulation. In this paper, we analyze the case of a six-turn series coil.

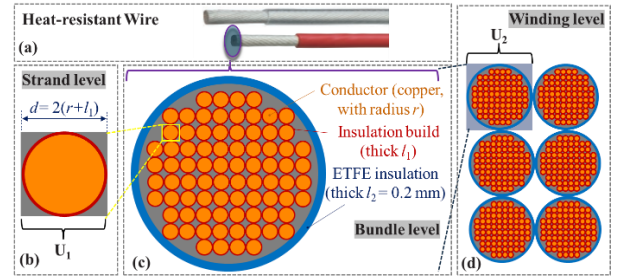


Fig. 1. Illustration of the 6-turn coil made of Litz wire. (a) Physical diagram of a bundle wire. (b) Strand level. (c) Bundle level. (d) Winding level.

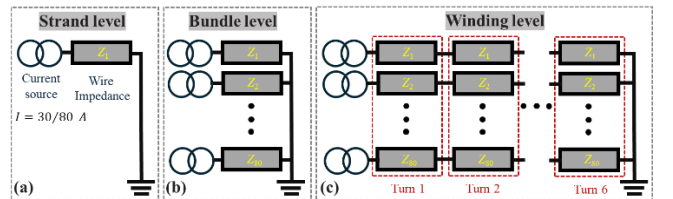


Fig. 2. Lumped element circuits. (a) Strand level. (b) Bundle level. (c) Winding level.

After finalizing the wire selection process, the analysis starts at the strand level shown in Fig. 1 (b). Defining twice the combined value of the single strand wire radius r and the isolation build thickness l_1 , the complex form of relative permeability $\langle \mu_r \rangle$ considering both Joule loss and the diamagnetic effect for the wire can be obtained from the extended Ollendorff formula [1]:

$$\langle \mu_r \rangle = 1 + \eta(\mu_r - 1) / [1 + N(1 - \eta)(\mu_r - 1)], \quad (1)$$

where η is the volume ratio of the wire over the squared cell U_1 with size d , N equals 0.5, and μ_r is the complex relative permeability of a stranded wire given by

$$\mu_r = \mu_r J_1(z) / [z J_0(z) - J_1(z)], \quad (2)$$

where J_0 and J_1 are the 0-th and 1st order Bessel, μ_r is the relative permeability of strand, and $z = (1 - j)r/\delta$. Using (1), the stranded wire is approximated to be a uniform cell U_1 with $\langle\mu_r\rangle$. Note that U_1 can consist of not only a single wire, but also multiple wires.

When these 80 cells are sequentially arranged to form a circular cross-section resembling a mosaic, they make up the homogenized coil of a single turn at the bundle level. The radius of the circle, designated by r_2 , is about 1.128 mm. Then the magnetic stored energy W_b and Joule losses J_b of the coil at this level can be obtained by the magnetostatic field analysis using finite element method with (1). According to [1], the aforementioned quantities are obtained by

$$W_b = \frac{1}{2} \int \text{Re}(\langle\mu_r\rangle) |\mathbf{H}|^2 d\Omega \quad (3)$$

$$J_b = -\omega \int \text{Im}(\langle\mu_r\rangle) |\mathbf{H}|^2 d\Omega + \text{Re}(Z_{skin}), \quad (4)$$

where Ω denotes computational domain, $\langle\mu_r\rangle = 1$ in air, $\mathbf{H}$ is the magnetic field, and Z_{skin} is the impedance caused by the skin effect [1].

Then we consider the winding level. We replace $\langle\mu_r\rangle$ for the strand level with the equivalent complex relative permeability $\langle\hat{\mu}_r\rangle$ for the bundle-level cell U_2 with the size of $2 \times (r_2 + l_2)$, as shown in Fig. 1 (c). Then $\langle\mu_r\rangle$ on the right-hand side in (3) and (4) is once again replaced by $\langle\hat{\mu}_r\rangle$ for the scale of winding.

III. COMPARISONS OF RESULTS

At the strand level, the real and imaginary parts for a stranded wire over the normalized wire radius (equal to r/δ) are shown in Fig. 3. The results by FEM are also provided based on [4]

$$\langle\mu_r\rangle = \int |\mathbf{B}_0|^2 / \mu_0 d\Omega / \left[\int \mu_r \mu_0 |\mathbf{H}|^2 d\Omega - \int \sigma |\mathbf{E}|^2 / j\omega d\Omega \right] \quad (5)$$

where $\mathbf{B}_0$, and $\mathbf{E}$ are the uniform magnetic flux density in the static field, and the electrical field in eddy currents respectively, μ_r and σ is the relative permeability and the conductivity of the wire, and μ_0 is the permeability in vacuum. It can be observed that the relative permeability calculated using (1) matches very well with the FEM results for both real and imaginary parts, with the maximum relative errors being $1.22 \times 10^{-3}\%$ and $1.25 \times 10^{-2}\%$, respectively. It is worth noting that the accuracy remains excellent even at 1 MHz.

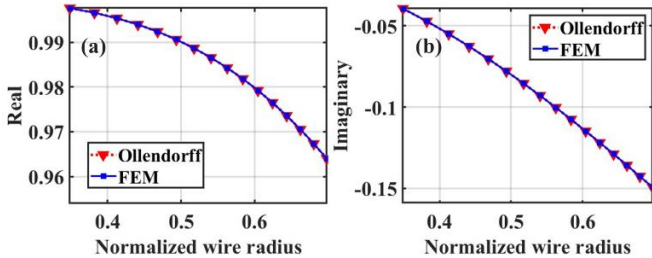


Fig. 3. Complex relative permeability $\langle\mu_r\rangle$ of the wire over the normalized wire radius r/δ at the strand level. (a) The real part. (b) The imaginary part.

At the bundle level, W_b and J_b of a turn are plotted in Fig. 4. For comparative validation, corresponding results computed using the full model of the 80-strand parallel twisted wire in the eddy current field are also presented. The maximum relative errors for W_b and J_b are 0.267% and 1.216% respectively, still indicating their high accuracy.

At the winding level, results of W_b and J_b by the proposed method and by the full-scale model are given in Fig. 5. From comparison, the maximum relative errors are 3.098% and 7.126%, respectively. Compared to the first two levels, the errors are larger but acceptable. There are two main reasons for the increase in errors: firstly, the example involves a relatively small number of turns; secondly, it stems from the accumulation of errors at each hierarchical level. Note that we can easily handle a greater number of turns at the winding level with the proposed homogenization method, although the conventional FE analysis is difficult for such cases.

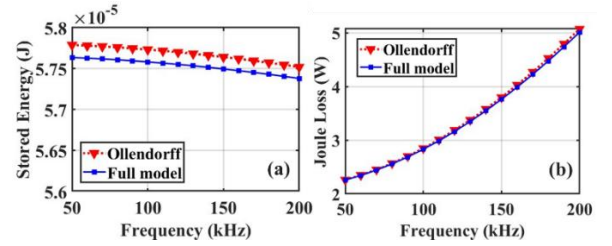


Fig. 4. Stored energy and Joule loss of the wire at the bundle level. (a) Stored energy. (b) Joule loss.

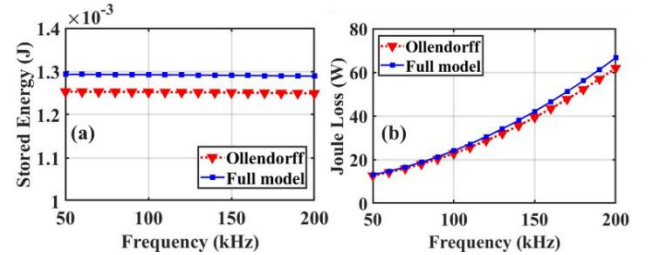


Fig. 5. Stored energy and Joule loss of the wire at the winding level. (a) Stored energy. (b) Joule loss.

IV. CONCLUSION

The multiscale semi-analytical homogenization method, which performs magnetostatic field analysis without fine discretization at each level, has been proved to be accurate on simplifying Litz wire analysis in frequency domain. In the full paper, this approach will be detailed for the cases where litz wire are stacked in the square and the hexagonal mode respectively.

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