

A Hybrid Subgridding Method Based on FDTD and SS2-PITD for Solving Multi-scale Problems

Mingjun Chi¹, Xikui Ma¹, Jiawei Wang¹, Shuli Yin¹, and Xiaojie Zhu²

¹School of Electrical Engineering, Xi'an Jiaotong University, Xi'an, China, 3120104253@stu.xjtu.edu.cn

²Dipartimento di Elettronica, Informazione e Bioingegneria, Politecnico di Milano, Milan, Italy, xiaojie.zhu@polimi.it

Based on the finite difference time domain (FDTD) method and the three-stage split-step precise integration time domain method (SS2-PITD), a hybrid subgridding method is proposed for solving multi-scale problems. In this proposed method, SS2-PITD is applied to analyze the small-scale fine structure with fine cells, while the FDTD method is employed for the extensive background region with coarse cells. Since SS2-PITD breaks the Courant–Friedrichs–Lewy (CFL) condition, the time step sizes in the fine and coarse cell regions can be synchronized, thus reducing the number of updating steps in the fine cell region and improving the computational speed. Numerical experiments are simulated to show the effectiveness of the proposed method.

Index Terms—Finite difference time domain (FDTD) method, hybrid method, subgridding, three-stage split-step precise integration time domain (SS2-PITD) method.

I. INTRODUCTION

The finite difference time domain (FDTD) method [1], proposed based on the Yee's cells and Maxwell's equations, is a widely used tool for solving electromagnetic (EM) problems. When modeling multi-scale problems, the FDTD method suffers from the high computational burden imposed by the uniform fine cell discretization. To solve this challenge, some subgridding FDTD methods are proposed. By locally refining the fine structures, these subgridding FDTD methods can efficiently save the computational resources. However, the subgridding FDTD method still needs to satisfy the Courant–Friedrichs–Lewy (CFL) condition and is limited by the small cell size.

In response, the precise integration time domain (PITD) method [2], [3], [4] is developed to break the CFL condition and achieves a good performance on the dispersion. However, the conventional PITD method faces the issue of high memory costs due to the column-vector arrangement of the field components and the non-discretization of the time derivative. To address this issue, a three-stage split-step PITD (SS2-PITD) [5] method, which transforms the original 2-D or 3-D problems into 1-D problems, is presented to significantly reduce the memory cost.

Here, a hybrid subgridding method is proposed by merging the SS2-PITD method with the switched-Huygens-subgridding FDTD method [6]. The SS2-PITD method is employed in the fine structures with fine cells, while the FDTD method is used in the rest region with coarse cells. As SS2-PITD breaks the CFL condition, the fields in the fine and coarse cell regions can be synchronously updated, which greatly reduces the number of updating steps in the fine cell region and thus achieves high efficiency.

II. FORMULATIONS

A. FDTD Method Used in Coarse Cell Region

For a linear homogeneous medium with permittivity ϵ and permeability μ , Maxwell's curl equations for the TE wave are

depicted as

$$\frac{\partial}{\partial t} \begin{bmatrix} E_x \\ E_y \\ H_z \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{\partial}{\epsilon \partial y} \\ 0 & 0 & -\frac{\partial}{\epsilon \partial x} \\ \frac{\partial}{\mu \partial y} & -\frac{\partial}{\mu \partial x} & 0 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ H_z \end{bmatrix}. \quad (1)$$

Both the temporal and spatial derivatives in (1) are approximated by the central differences. Then, (1) is reformulated as the updating formula for the FDTD method, depicted as

$$(\mathbf{I} - \mathbf{G}\Delta t) \begin{bmatrix} E_x^{n+1} \\ E_y^{n+1} \\ H_z^{n+\frac{3}{2}} \end{bmatrix} = (\mathbf{I} + \mathbf{F}\Delta t) \begin{bmatrix} E_x^n \\ E_y^n \\ H_z^{n+\frac{1}{2}} \end{bmatrix} \quad (2)$$

where n denotes the time index, Δt is the time step size, $\mathbf{I}$ is the unit matrix. $\mathbf{G}$ and $\mathbf{F}$ are expressed as

$$\mathbf{G} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{\mathbf{D}_y}{\mu} & -\frac{\mathbf{D}_x}{\mu} & 0 \end{bmatrix}, \mathbf{F} = \begin{bmatrix} 0 & 0 & \frac{\mathbf{D}_y}{\epsilon} \\ 0 & 0 & -\frac{\mathbf{D}_x}{\epsilon} \\ 0 & 0 & 0 \end{bmatrix}. \quad (3)$$

$\mathbf{D}_\alpha (\alpha = x, y)$ arise from the discretization of $\frac{\partial}{\partial \alpha}$.

B. SS2-PITD Method Used in Fine Cell Region

For simplicity, (1) is rewritten as

$$d\mathbf{X}/dt = \mathbf{A}\mathbf{X} + \mathbf{B}\mathbf{X} \quad (4)$$

where $\mathbf{X} = [E_x, E_y, H_z]^T$, $\mathbf{A} = \text{tril}(\mathbf{G} + \mathbf{F})$ and $\mathbf{B} = \text{triu}(\mathbf{G} + \mathbf{F})$. The functions "tril" and "triu" have the same usage as in MATLAB.

Considering the three-stage split-step scheme in [5], the solution to (4) can be obtained by successively solving the following sub-steps with time increments $\Delta t/4$, $\Delta t/2$, and $\Delta t/4$, respectively.

$$\text{sub-step1 } d\mathbf{X}/dt = 2\mathbf{A}\mathbf{X} \quad (5a)$$

$$\text{sub-step2 } d\mathbf{X}/dt = 2\mathbf{B}\mathbf{X} \quad (5b)$$

$$\text{sub-step3 } d\mathbf{X}/dt = 2\mathbf{A}\mathbf{X}. \quad (5c)$$

Due to the limited space, the precise integration (PI) technique for solving (5a)-(5c) can be found in [2], not shown here.

C. Connection Between FDTD and SS2-PITD

The information exchange between FDTD and SS2-PITD is realized by the Huygens surfaces, referred to as the inner surface (IS) and the outer surface (OS). The concept of the Huygens surface originates from the equivalence theorem. The theorem states that, the EM wave can be radiated from region A to region B by imposing the following electric and magnetic current densities on the surface separating regions A and B:

$$\mathbf{J}_s = \mathbf{n} \times \mathbf{H}_s \quad (6a)$$

$$\mathbf{K}_s = -\mathbf{n} \times \mathbf{E}_s \quad (6b)$$

where the surface is called the Huygens surface. $\mathbf{n}$ is the Huygens surface's unit normal vector with the orientation from region A to region B. $\mathbf{E}_s$ and $\mathbf{H}_s$ are the magnetic and electric fields sampled on the Huygens surface in region A.

Fig. 1 shows how to implement the Huygens surfaces IS and OS, and is drawn with $n_s = \frac{\Delta x_a}{\Delta x_b} = \frac{\Delta y_a}{\Delta y_b} = 2$, where $\Delta\alpha$ ($\alpha = x, y$) is the spatial step size on the α -direction. The subscripts a and b are used to distinguish variables in FDTD and SS2-PITD, respectively. In the hybrid subgridding method, OS radiates the EM wave from FDTD to SS2-PITD, and IS radiates the EM wave from SS2-PITD to FDTD.

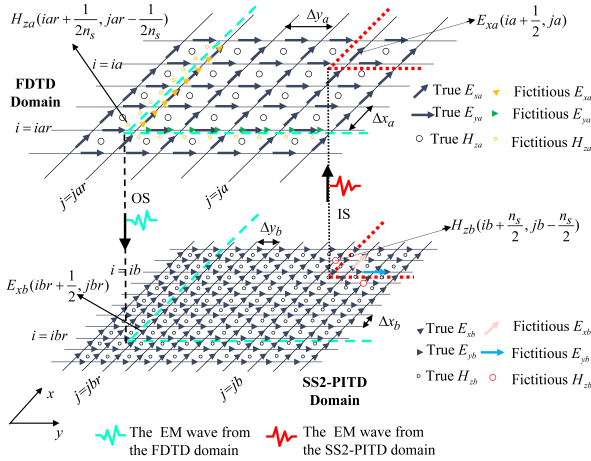


Fig. 1. The implementation of IS and OS.

The implementation of IS and OS can use the discrete interpretation of the total field/scattered field (TF/SF) boundary. Thus, the fields adjacent to IS and OS need to be modified with the EM wave compensation from SS2-PITD and FDTD, respectively. For example, in the implementation of IS, the modified updating formula for $E_{xa}(ia + \frac{1}{2}, ja)$ in Fig. 1 is depicted as

$$\begin{aligned} E_{xa}^{n+1}(ia + \frac{1}{2}, ja) &= E_{xa}^n(ia + \frac{1}{2}, ja) + \frac{\Delta t}{\epsilon \Delta y} [\\ &H_{za}^{n+\frac{1}{2}}(ia + \frac{1}{2}, ja + \frac{1}{2}) - H_{za}^{n+\frac{1}{2}}(ia + \frac{1}{2}, ja - \frac{1}{2}) \\ &+ H_{zb}^{n+\frac{1}{2}}(ib + \frac{n_s}{2}, jb - \frac{n_s}{2})] \end{aligned} \quad (7)$$

where $H_{zb}^{n+\frac{1}{2}}(ib + \frac{n_s}{2}, jb - \frac{n_s}{2})$ is the average of $H_{zb}^{n+\frac{1}{2}}(ib + \frac{1}{2} : ib + n_s - \frac{1}{2}, jb + \frac{1}{2} - n_s : jb - \frac{1}{2})$. ":" has the same usage as MATLAB.

III. NUMERICAL EXPERIMENTS

To show the effectiveness of the proposed hybrid subgridding method, a ground penetrating radar case, shown in Fig. 2(a), is simulated. In this case, the detection target is a small-scale PEC fine structure buried in the medium ($\epsilon_r = 9$, $\mu_r = 1$). The transmitting antenna and receiving antenna (referred to as T and R, respectively) are located at 4.5 mm from the medium. Besides, T is 9 mm away from R. T emits a signal $I_m(t) = \exp(-(t - t_0)^2/\tau^2)$, where τ is 1×10^{-10} , and t_0 is 4.5×10^{-10} . After propagation and reflection, the signal is captured by R. In the hybrid subgridding method, the region surrounding the PEC fine structure is analyzed by the SS2-PITD method, and the rest is analyzed by the FDTD method. The coarse cell size used in the FDTD domain is $\Delta x_a = \Delta y_a = \Delta z_a = 3$ mm, while the SS2-PITD domain is refined by $n_s = 3$. For comparison, the FDTD method is performed with fine and coarse cells, respectively.

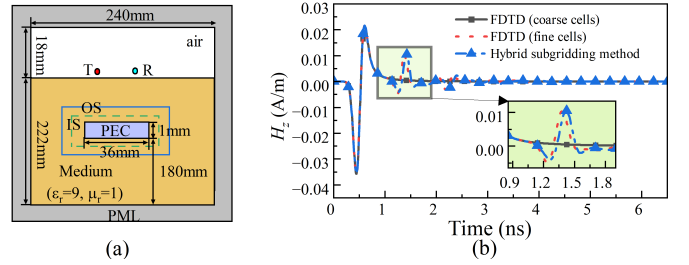


Fig. 2. (a) The structure of the ground-penetrating radar system. (b) The fields sampled at R.

The simulation results are shown in Fig. 2(b), and it can be seen that the FDTD method with coarse cells is insufficient to accurately deal with the fine structure, indicating the necessity of fine cell discretization. Since the local refinement of the PEC fine structure, the result of the hybrid subgridding method is consistent with the FDTD method with fine cells, which shows the reliability and effectiveness of the hybrid subgridding method. In addition, the simulation time of the hybrid subgridding method is 10.01 s, which is only 33.61 % of the FDTD method with fine cells, significantly improving the computational speed.

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