

Reduced Order Hysteretic Magnetization and Flux Models for Composite Superconductors

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The electric and magnetic response of type-II irreversible superconductors to external field and transport current is hysteretic due to magnetic flux pinning in the material. This hysteretic behavior can be accurately reproduced in finite element models with a nonlinear electrical resistivity, but it leads to computationally expensive simulations. Consequently, this approach is unrealistic for large-scale systems such as superconducting accelerator magnets which are multi-scale structures with a cross section containing hundreds or thousands of superconducting strands. In this contribution, we present reduced order methods based on an energy-based vector hysteresis model to describe the macroscopic response of composite superconductors without a need to compute the detailed current density distribution. The model also reproduces rate-dependent effects due to eddy currents and coupling currents in normal conducting parts of the composite.

Index Terms—Type-II superconductors, reduced order modelling, finite element analysis, ac loss, magnetization.

I. INTRODUCTION

SUPERCONDUCTING strands and cables under time-varying magnetic field or current generate loss, field distortion, and differential inductance that result from an interplay between hysteresis, eddy, inter-filament coupling, and inter-strand coupling currents [1]. The response of a single strand can be reliably computed with the finite element (FE) method and detailed strand models [2], using a nonlinear law for the electrical resistivity, e.g., the power law [3] with a typical exponent value, typically between 20 and 100. Using a similar approach to model large-scale systems such as accelerator magnets whose cross section contains hundreds or thousands of strands is challenging, as it leads to prohibitive simulation times which are unachievable in practice [4]. However, accounting for the effect of small-scale dynamics in strands is essential, because it affects the cryogenic heat load as well as the temperature and stability margins of the magnets [5], [6]. There is therefore a need for reduced order models, which reproduce the macroscopic effects of the small-scale dynamics without computing the current density distribution in detail.

In this contribution, we present the Reduced Order Hysteretic Magnetization (ROHM) model [7] and the Reduced Order Hysteretic Flux (ROHF) model. These models are inspired by the energy-based vector hysteresis model for ferromagnetic materials [8], [9], [10], and adapted to superconductors to account for hysteresis, eddy, and coupling effects.

In the following sections, we briefly introduce both models and illustrate their application to a multifilamentary superconducting strand. More generally, the models can be used and will be adapted to other types of superconductors, such as Rutherford cables or high-temperature superconducting tapes.

II. ROHM AND ROHF MODELS

We consider a superconducting strand such as represented in Fig. 1. It consists of many superconducting filaments, e.g., Nb-Ti, Nb₃Sn, or MgB₂, embedded and twisted in a normal

conducting matrix, e.g., Cu. The strand is subject to a time-varying magnetic field $\mathbf{h}(t)$ and a transport current $I(t)$. We analyze these two excitations separately, and model their effect with the ROHM and ROHF models, respectively.

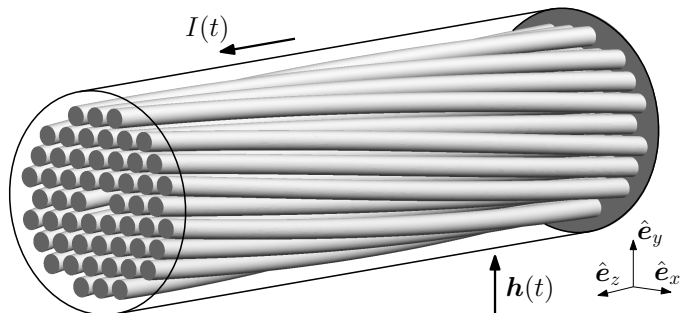


Fig. 1. Composite strand with twisted superconducting filaments embedded in a conducting matrix (cylindrical outline) subject to a time-varying transverse magnetic field and transport current.

A. ROHM model for applied field

The external magnetic field $\mathbf{h}$ generates a current density distribution $\mathbf{j}$. This generates a magnetization $\mathbf{m}$ that distorts the surrounding field and may store and dissipate energy. As shown in Fig. 2, the magnetization is hysteretic, due to the electric response of the superconducting filaments, and rate-dependent, because of coupling currents between filaments [1] as well as eddy currents in the normal conducting matrix.

The ROHM model describes the magnetization $\mathbf{m}$ directly in terms of the magnetic field $\mathbf{h}$ by defining a hysteretic constitutive law for the magnetic flux density $\mathbf{b} = \mathbf{b}(\mathbf{h})$, with $\mathbf{m} = \mathbf{b} - \mu_0 \mathbf{h}$ ($\mu_0 = 4\pi \times 10^{-7}$ H/m). The constitutive law is defined as a chain of simple hysteretic elements connected in series and with fitting parameters based on reference solutions [7].

A key feature of this method is that it provides separate access to dissipated and stored magnetic energies at every time instant, and not only after closing a magnetization loop [7], [9]. It also allows for the description of minor hysteresis loops.

Finally, it can be easily implemented in FE models, in which the detailed strand cross-sections (see Fig. 1) can be replaced by a homogeneous non-conducting region behaving as a magnetic material with $\mathbf{b} = \mathbf{b}(\mathbf{h})$ as a constitutive law.

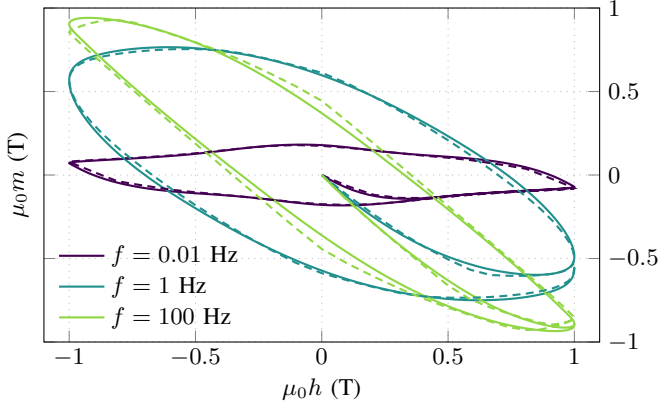


Fig. 2. Average strand magnetization for an applied magnetic field varying sinusoidally with an amplitude of 1 T and frequency f . Solid curves are reference results from [2]. Dashed curves are results from the ROHM model.

B. ROHF model for transport current

A net transport current I generates a current density distribution $\mathbf{j}$. It has a zero net magnetization but it defines a magnetic flux Φ whose dependence on I is also hysteretic, as shown in Fig. 3. The internal flux Φ stores magnetic energy and may also dissipate it with composite superconductors. The time derivative of the internal flux is equal to the electrical voltage applied on the strand.

Similarly to the ROHM model, the ROHF model describes the internal flux Φ directly in terms of the transport current I , via a scalar hysteretic relationship $\Phi = \Phi(I)$. This model has the same theoretical foundation as the energy-based hysteresis model for ferromagnetic materials [8], [9] and therefore also clearly separates stored and dissipated energy quantities.

The ROHF model can be used in FE models to describe the voltage-current relationship of current-carrying superconducting strands, which can be modelled as homogeneous conductors with uniform current density, as is done for a stranded conductor [11].

III. RESULTS AND OUTLOOK

Results of the ROHM model are shown in Fig. 2 for a chain of 5 hysteretic elements. The model reproduces the uncoupled filament regime at low frequency as well as coupled filaments regime at higher frequency [1], with gradually increasing magnetization. The ROHM model can be tailored to produce results with a desired accuracy by increasing or reducing the number of hysteretic elements.

Results for the ROHF model are given in Fig. 3 with a chain of 7 cells. Work is in progress to include rate-dependent effects in the ROHF model.

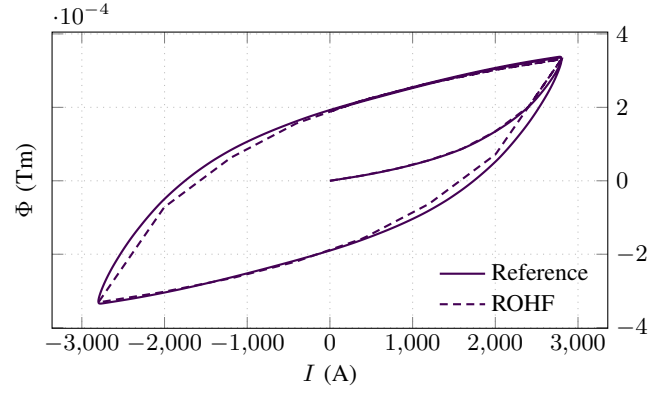


Fig. 3. Strand internal flux per unit length for a transport current varying sinusoidally with amplitude of 2.8 kA and a frequency of 0.1 Hz. Solid curves are reference results from [2]. Dashed curves are results from the ROHF model.

In the final contribution, we will provide a detailed analysis of the capabilities and limitations of both models, in terms of accuracy and numerical efficiency. We will also discuss the combination of the two models in order to handle cases involving both applied field and transport current excitations.

At the conference, the inclusion of the ROHM and ROHF models in a full-scale magnet model will be presented in another contribution [12].

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