

A Coulomb Gauged Multi-Scale Finite Element Method Based on Magnetic Vector Potential in Laminated Iron Cores

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The traditional multi-scale finite element method (FEM) used to solve the lamination eddy current problem does not define the gauge condition of magnetic vector potential, which leads to numerical instability problems such as excessive residuals in the corresponding equations during the solution process. In this paper, a Coulomb gauged multi-scale FEM is proposed for the eddy current problem in lamination iron cores, which ensures the solution uniqueness of magnetic vector potential by weakly defining the global divergence of the potential function, thereby improving the numerical stability of the FEM solutions. The benchmark model TEAM 21d is solved by using the proposed method, and the comparison of calculated results and measured ones show that the method can effectively and accurately solve the eddy current problem in the laminated cores.

Index Terms—Laminated iron cores, Multi-scale finite element method, Coulomb Gauge, Eddy current problems

I. INTRODUCTION

For the larger power transformers, laminated core structures are commonly used to reduce the losses of eddy current. The dimensions of the laminated core typically exhibit a two-scale property, with the width and height being approximately 10^6 times greater than the thickness. Therefore, accurately simulating the laminated cores with normal FEM requires highly refined mesh divisions, which results in significant computational costs. In order to minimize computational costs and ensure simultaneously the precision of the numerical solutions, special numerical simulation methods for laminated iron cores has become a hot point of research. The eddy currents in the laminated core can be classified into two types: the larger eddy currents generated by vertical leakage magnetic flux and the smaller eddy currents generated by parallel main magnetic flux. Some scholars have made some creative works on homogenization methods and embedding low dimensional model^[1,2]. However, these methods do not model in detail the small eddy current loops caused by the main magnetic flux and the magnetic flux distribution in the laminated core. Multi-scale finite element methods (MSFEMs) can effectively reduce computational costs by extracting the eddy current distribution in semi-infinite conductive plates and establishing microscopic expressions to represent the eddy currents on each lamination^[3]. However, traditional multi-scale finite element methods do not account for the solution uniqueness of the magnetic vector potential, which leads to numerical instability during the solution process of FEM equations.

To address the above problem of numerical instability, a method is proposed of applying Coulomb gauge conditions in multi-scale finite element method based magnetic vector potential A . The efficiency of the method is validated by using the TEAM 21d benchmark problem. The comparison of calculated results and measured ones shows that the proposed method significantly improves the numerical stability of multi-scale FEM.

II. 3D MULTI-SCALE FINITE ELEMENT METHOD WITH COULOMB GAUGE FOR LAMINATED IRON

The model of eddy current problem in laminated iron is depicted in Fig.1. The computational domain Ω consists of the conducting domain Ω_1 and the surrounding domain Ω_2 containing the excitation source with the boundary $\Gamma = \Gamma_B \cup \Gamma_H$. The corresponding boundary value problem based on the modified magnetic vector potential (MVP) A is:

$$\begin{aligned} v\nabla \times \nabla \times A + \sigma \frac{\partial A}{\partial t} &= J \\ n \times A &= 0 \quad \text{on } \Gamma_B \\ v\nabla \times A \times n &= 0 \quad \text{on } \Gamma_H \end{aligned} \quad (1)$$

where v is the magnetic reluctivity, σ the electric conductivity, J the given current density.

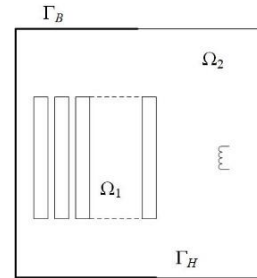


Fig. 1. Model of eddy current problem in laminated iron $\Omega = \Omega_1 \cup \Omega_2$.

A. Traditional multi-scale finite element formulation

By extracting the eddy current distribution characteristics of the semi-infinite conductor plate, vector potentials A_0 , A_1 and scalar function w are introduced and the multi-scale FEM formulation is as below:

$$A = A_0 + \phi A_1 + \nabla(\phi w) \quad (2)$$

A_0 in (1) characterizes the solution on a large scale, ϕA_1 the eddy current flowing parallel to the laminates and $\nabla(\phi w)$ the edge effect. The micro-shape scalar function ϕ is linearly related to the width of one laminate d and the gaps d_0 between the laminates. Choosing a suitable finite element function space A_0

$\in H(\text{curl}, \Omega)$, $A_1 \in H(\text{curl}, \Omega_1)$, $w \in H^1(\Omega_1)$ according to the de-Rham complex.

B. Incorporation of Gauge condition

According to the Helmholtz theorem of vector analysis, the vector potential A will be unique when its curl and divergence are defined. While in (1), only the curl of A is defined. Although the divergence of the edge element basis is zero within each element, the jumps of its normal component between elements may lead to a globally large weak divergence.

The divergence of vector potential A in conducting area Ω_1 is zero because of the eddy current continuity. A scalar variable χ is introduced to define the divergence of A_0 in surrounding area Ω_2 . The field equations are proposed :

$$\begin{aligned} \nabla \times \nabla \times A + \sigma \frac{\partial A}{\partial t} - \nabla \chi &= J \\ \nabla \cdot A + \chi &= 0 \quad \text{in } \Omega_2 \end{aligned} \quad (3)$$

The current density J in (3) should satisfy the current density continuity so that the compatibility of the equation can be guaranteed seriously. χ satisfies the Laplace equation taking the divergence of the first equation in (3). It can be found that χ is zero when the homogeneous Dirichlet boundary condition for χ is enforced on the boundary of Ω_2 .

The weak formulation is :

Find $(A_{0h}, A_{1h}, w_h, \chi_h) \in V_{h,0} := \{ (A_{0h}, A_{1h}, w_h, \chi_h) : A_{0h} \in U_h, A_{1h} \in V_h, w_h \in W_h \text{ and } \chi_h \in Y_h \text{ and } A_0 \times n = 0 \text{ on } \Gamma \}$, such that

$$\begin{aligned} \int_{\Omega} \nabla \times N_h \cdot \nabla \times A_h d\Omega + \frac{\partial}{\partial t} \int_{\Omega} \sigma N_h \cdot A_h d\Omega - \int_{\Omega} N_h \cdot \nabla \chi_h d\Omega \\ = \int_{\Omega} N_h \cdot J d\Omega \\ - \int_{\Omega} \nabla N_h \cdot A_h d\Omega + \int_{\Omega} N_h \cdot \chi_h d\Omega = 0 \end{aligned} \quad (4)$$

for all $(N_{0h}, N_{1h}, q_h, r_h) \in V_{h,0}$, where U_h is an FE subspace of $H(\text{curl}, \Omega)$, V_h of $H(\text{curl}, \Omega_1)$, W_h of $H^1(\Omega_1)$ and X_h of $H^1(\Omega_2)$. Dirichlet boundary conditions are prescribed for A_0 and χ on Γ , and homogenous natural boundary conditions hold for w and A_1 in Γ_{12} .

III. RESULTS AND DISCUSSIONS

TEAM workshop problem 21d is used to test the accuracy of 3D nonlinear eddy current and hysteresis analysis model with anisotropic lamination without solid magnetic steel. This problem consists of 20 Silicon laminations and two exciting coils. Currents in opposite directions are applied to the two coils, and the main magnetic flux generated is perpendicular to the direction of the lamination. The structure is shown as Fig.2.

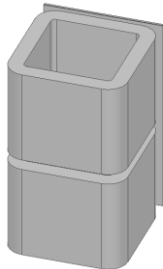


Fig. 2. Structure of TEAM 21d.

Table 1 shows the comparison between the MSFEM calculation results and the measurement results of the 21d model under different excitation currents.

TABLE I
COMPARISON BETWEEN MSFEM AND MEASURED RESULTS

Exciting Currents (A, rms, 50Hz)	MSFEM(W)	Measured(W)
10	3.20	2.20
15	7.19	5.30
20	12.79	10.20
25	19.98	16.80

Figure 3 shows the calculated results of leakage flux in the air region using multi-scale finite element method under 25A current excitation.

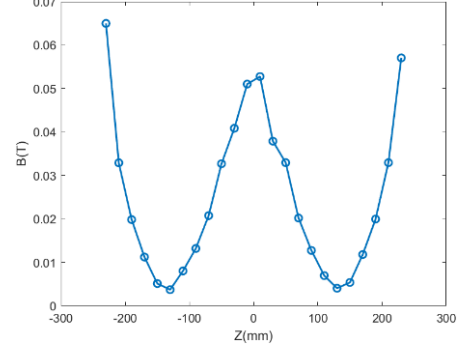


Fig. 3. Calculated results of leakage flux at x=3.76mm,y=0.

Since the saturation problem of magnetic materials and the anisotropy of oriented silicon steel sheets are not considered, the above results have certain errors. Subsequent work will be devoted to solving these problems.

IV. CONCLUSION

A multiscale FEM formulation with Coulomb gauge for the modified MVP is presented. In conduction region, the divergence of MVP is implicitly satisfied due to the continuity of eddy current. The Coulomb gauge is enforced by introducing a scalar variable χ for nonconducting region. The proposed method results in symmetric matrix system and can be solved by direct solvers robustly. TEAM 21d model is presented to show the accuracy of the proposed method for the engineering lamination eddy current problems.

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REFERENCES

- [1] B  r  , Oszk  r, Kurt Preis, and Igor T  car. "A FEM method for eddy current analysis in laminated media." *COMPEL-The international journal for computation and mathematics in electrical and electronic engineering* 24.1 (2005): 241-248.
- [2] Bottauscio, Oriano, Mario Chiamp  , and Daniela Chiarabaglio. "Advanced model of laminated magnetic cores for two-dimensional field analysis." *IEEE Transactions on Magnetics* 36.3 (2000): 561-573.
- [3] Hollaus, Karl. "A MSFEM to simulate the eddy current problem in laminated iron cores in 3D." *COMPEL-The international journal for computation and mathematics in electrical and electronic engineering* 38.5 (2019): 1667-1682.
- [4] Zhao, Yanpu, and W. N. Fu. "A novel Coulomb-gauged magnetic vector potential formulation for 3-D eddy-current field analysis using edge elements." *IEEE Transactions on Magnetics* 53.6 (2017): 1-4.