

Harmonic Balance Finite Element Method Combined With a Homogenization of Laminated Yokes

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The magnetoquasistatic simulation of laminated ferromagnetic structures at elevated frequencies is a challenging task, which quickly becomes impossible for three-dimensional models, especially if a nonlinear magnetization curve must be considered. In this contribution, we show how existing homogenization techniques for the linear case can be combined with the harmonic balance finite element method, to also account for the nonlinearity of the magnetization curve. This leads to a method which does not necessitate resolving the individual laminations with the finite element mesh and avoids time-stepping, thus allowing us to conduct nonlinear eddy current simulations of large three-dimensional models at frequencies in the kilohertz range efficiently. In particular, we demonstrate the usefulness of this new approach for simulating fast orbit corrector magnets in particle accelerators.

Index Terms—Accelerator magnets, Eddy currents, Finite element analysis, Nonlinear magnetics

I. INTRODUCTION

FAST orbit corrector magnets are an important component in the new generation of synchrotron radiation sources which produce extremely powerful X-rays and thus enable new experiments in diverse fields of science. The magnets are part of a control system which stabilizes the particle beam against various disturbances. To design this control system, the dynamic behavior of the magnets must be understood at frequencies up to the kilohertz range. To that end, finite element (FE) simulations are needed. However, simulating these magnets is challenging. Due to the laminated structure of the yokes and the small skin depths at elevated frequencies, a very fine mesh is necessary, leading to prohibitive simulation times. For the linear case, homogenization techniques can mitigate this issue. For the nonlinear case, however, there is no adequate method for simulating models of this size. Therefore, we have developed a dedicated method combining homogenization techniques for lamination stacks with the harmonic balance finite element method (HBFEM). This combination, here called HomHBFEM, allows us to simulate the laminated magnets with a relatively coarse mesh and a nonlinear B - H curve without time-stepping.

II. METHOD

A. Harmonic Balance Finite Element Method

The transient nonlinear magnetoquasistatic problem reads

$$\nabla \times \left(\nu(t) \nabla \times \vec{A}(t) \right) + \sigma \frac{\partial \vec{A}(t)}{\partial t} = \vec{J}_s(t), \quad (1)$$

where $\vec{A}$ is the magnetic vector potential, $\vec{J}_s$ is the applied current density, σ denotes the conductivity and ν the reluctivity. The latter is a function of the time t since its value depends on the magnetic flux density $\vec{B}(t) = \nabla \times \vec{A}(t)$. For brevity, we do not indicate the spatial dependence explicitly. Transforming (1) into the frequency domain yields

$$\nabla \times \left(\underline{\nu}(\omega) * \nabla \times \vec{A}(\omega) \right) + j\omega\sigma\vec{A}(\omega) = \vec{J}_s(\omega), \quad (2)$$

where $*$ is the convolution operator. The HBFEM formulation is obtained from (2) by truncating the convolution after a certain maximum order of the considered harmonics, $m \in \mathbb{N}$, and discretizing according to the FE method (FEM) [1]. This leads to a nonlinear system of equations

$$\sum_{k=\max\{-m, n-m\}}^{\min\{m, n+m\}} \mathbf{K}_{\nu_k}(\underline{\hat{a}}) \underline{\hat{a}}_{n-k} + j\omega_f \mathbf{M}_\sigma \underline{\hat{a}}_n = \underline{\hat{j}}_{s,n} \quad (3)$$

for $n \in \mathbb{Z} \cap [-m, m]$. Herein, ω_f is the fundamental angular frequency, $\underline{\hat{a}}_n$ is the vector gathering the degrees of freedom (DoFs) of the n -th harmonic of the magnetic vector potential, $\underline{\hat{j}}_{s,n}$ is the discretized n -th harmonic of the applied current density, $\mathbf{M}_\sigma$ is the mass matrix, and $\mathbf{K}_{\nu_k}$ denotes the stiffness matrix computed with the k -th harmonic of the reluctivity. We indicate that each harmonic of the reluctivity depends on all the harmonics of the magnetic vector potential by using the notation $\mathbf{K}_{\nu_k}(\underline{\hat{a}})$ [2]. Considering $\underline{\hat{a}}_{-n} = \underline{\hat{a}}_n^*$, we only have to solve for $(\underline{\hat{a}}_n)_{n \in \mathbb{N}_0}$, which we do via successive substitution.

B. Homogenization Techniques

In the laminated yokes of the magnets, ν and σ are functions of the spatial coordinate, since they are different for the conducting laminates and their isolating coating. The homogenizations used in this work replace ν and σ inside the laminated structure with spatially constant material tensors $\bar{\sigma}$ and $\bar{\nu}$. Herein, the reluctivity tensor is frequency-dependent. Examples of such techniques can be found in [3] and [4].

C. Combination

We implemented a combination of such homogenizations with the HBFEM in the FE software GetDP [5]. The crucial point is the incorporation of the frequency-dependent reluctivity tensors in the HBFEM. To that end, (3) is modified and becomes

$$\mathbf{K}_{\bar{\nu}_0(n\omega_f)}(\hat{\underline{a}})\hat{\underline{a}}_n + \sum_{k=\max\{-m, n-m\}, k \neq 0}^{\min\{m, n+m\}} \mathbf{K}_{\nu_k}(\hat{\underline{a}})\hat{\underline{a}}_{n-k} + jn\omega_f \mathbf{M}_{\bar{\sigma}}\hat{\underline{a}}_n = \hat{\underline{j}}_{s,n}, \quad (4)$$

where $\mathbf{M}_{\bar{\sigma}}$ is the mass matrix using the conductivity tensor $\bar{\sigma}$ of the homogenization and $\mathbf{K}_{\bar{\nu}_0(n\omega_f)}$ is the stiffness matrix constructed with the reluctivity tensor evaluated at $\omega = n\omega_f$, using the DC-component ν_0 for each element.

III. VERIFICATION

We verified the HomHBFEM by comparing our results to those of transient simulations conducted with CST Studio Suite® [6], where the laminations and the skin depth are resolved by the mesh. In the following, we show a brief excerpt of our verification studies with maximum order $m = 5$.

Figure 1 shows a simple inductor and Fig. 2 depicts the magnetic energies in its laminated core. We observe a good agreement of HomHBFEM with the transient CST simulation.

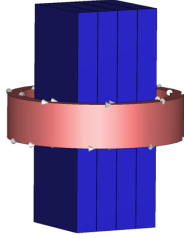


Fig. 1: Model of a simple inductor.

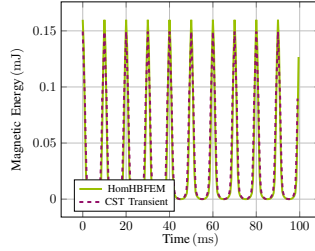


Fig. 2: Magnetic energy in the core.

Another investigated model is a C-magnet, see Fig. 3. Figure 4 shows the magnetic flux densities in the center of the aperture. Again, we observe good agreement with the transient CST simulation. Finally, Fig. 5 shows the convergence of the eddy current losses computed in CST at $f = 1$ kHz w.r.t. the number of DoFs in the model. We observe that as the mesh is refined, the losses computed in CST converge against the losses that we computed with the HomHBFEM with much less DoFs ($4.8 \cdot 10^4$). As a result, the CST simulation takes multiple days on a powerful computer with 192 GB of RAM and an Intel Xeon X5690 processor, while the HomHBFEM simulation can be done within a few hours on an ordinary laptop with 16 GB of RAM and an Intel Core i7 processor.

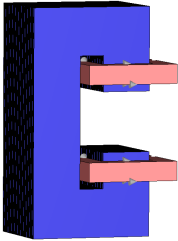


Fig. 3: Model of a C-shaped magnet.

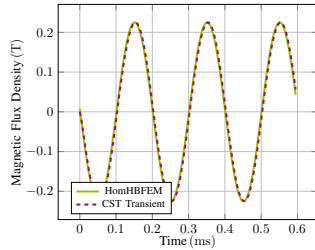


Fig. 4: Flux densities in the aperture center at $f = 5$ kHz.

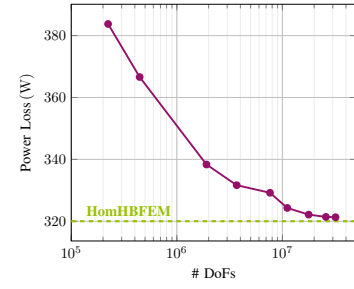


Fig. 5: Mesh dependence of eddy current losses in the C-magnet's yoke computed in CST Studio Suite® at $f = 1$ kHz. The green line indicates the result with HomHBFEM.

IV. APPLICATION

We applied the HomHBFEM to a model of an orbit corrector magnet, see Fig. 6. The model was provided by the particle accelerator laboratory Deutsches Elektronen-Synchrotron (DESY) and could not be simulated with commercial software.

Figure 7 shows the magnitude of the vertical component of the magnetic flux density along the axis obtained with our method for the linear and the nonlinear case. We can clearly see that the nonlinearity suppresses the integrated field along the axis. We have found that this effect plays a role with a lamination thickness of $d = 1$ mm, which is the lamination thickness of the prototype magnet, but becomes largely irrelevant for $d = 0.3$ mm, which is the currently favored option for series production.

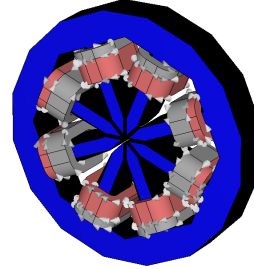


Fig. 6: Model of the fast orbit corrector magnet.

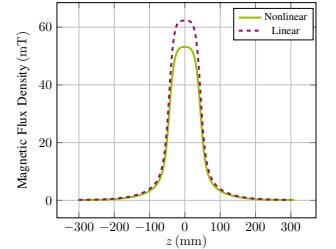


Fig. 7: Flux densities along the longitudinal axis at $f = 10$ kHz.

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