

An Effective Material Approach for the Nonlinear Eddy Current Problem for a Laminated Open Core Including Stray Fields

Markus Schöbinger¹, Stjepan Frlić², and Karl Hollaus¹

¹Technische Universität Wien, Institute of Analysis and Scientific Computing, 1040 Vienna, Austria
markus.schoebinger@tuwien.ac.at

²University of Zagreb, Faculty of Electrical Engineering and Computing, Unska 3, 10000 Zagreb, Croatia

Effective materials are an efficient way to solve the eddy current problem for devices consisting of many thin steel sheets. They allow to replace the fine-structured geometry by a bulk medium that is cheap to resolve in the finite element mesh. The parameters of this effective material are obtained using a suitable cell problem. By utilizing a complex permeability, the effective formulation becomes a static one, requiring only a magnetic scalar potential, which further reduces both the required memory and the computation time. However, conventional effective material theory requires the magnetic flux to be aligned with the sheets, i.e. stray fields cannot be accounted for. This paper presents a way to include stray fields in an effective material in an efficient way. Numerical experiments show very satisfying results at the cost of only a single additional scalar potential restricted to the bulk medium.

Index Terms—Eddy current, homogenization, nonlinear materials, stray fields

I. INTRODUCTION

OPEN laminated cores display two major electromagnetic properties: The main magnetic flux inducing narrow eddy current loops, and the stray field, which results in large current loops parallel to the sheets [1].

Effective material methods have been shown to accurately approximate the losses in closed cores due to the main magnetic flux, using drastically less computational resources than a full simulation [2]. However, they cannot account for the stray field. Similar to the method presented in [3], this work includes stray field effects into the effective material approach using an anisotropic formulation.

II. THE $\mathbf{T}$, Φ - Φ -FORMULATION

The magnetic field $\mathbf{H}$ is written as

$$\mathbf{H} = \mathbf{T} - \nabla \Phi + \mathbf{H}_{BS}. \quad (1)$$

The current vector potential $\mathbf{T}$ fulfilling $\text{curl} \mathbf{T} = \mathbf{J}$ with the current density $\mathbf{J}$ is defined only on the conducting domain Ω_c . The magnetic scalar potential Φ is defined on the entire domain $\Omega = \Omega_0 \cup \Omega_c$, and $\mathbf{H}_{BS}$ is the Biot-Savart field of the given currents.

This yields the weak formulation: Find $\mathbf{T} \in H(\text{curl}, \Omega_c)$ and $\Phi \in H^1(\Omega)$ so that

$$\begin{aligned} & \int_{\Omega_0} j\omega\mu_0 \nabla \Phi \cdot \nabla q \, d\Omega \\ & + \int_{\Omega_c} \rho \text{curl} \mathbf{T} \cdot \text{curl} \mathbf{v} + j\omega\mu(\mathbf{T} - \nabla \Phi) \cdot (\mathbf{v} - \nabla q) \, d\Omega \\ & = \int_{\Omega_0} j\omega\mu_0 \mathbf{H}_{BS} \cdot \nabla q \, d\Omega - \int_{\Omega_c} j\omega\mu \mathbf{H}_{BS} \cdot (\mathbf{v} - \nabla q) \, d\Omega \end{aligned} \quad (2)$$

for all $\mathbf{v} \in H(\text{curl}, \Omega_c)$ and $q \in H^1(\Omega)$, with the electric resistivity ρ , the magnetic permeability μ and the angular frequency ω . This formulation is used for the reference solution.

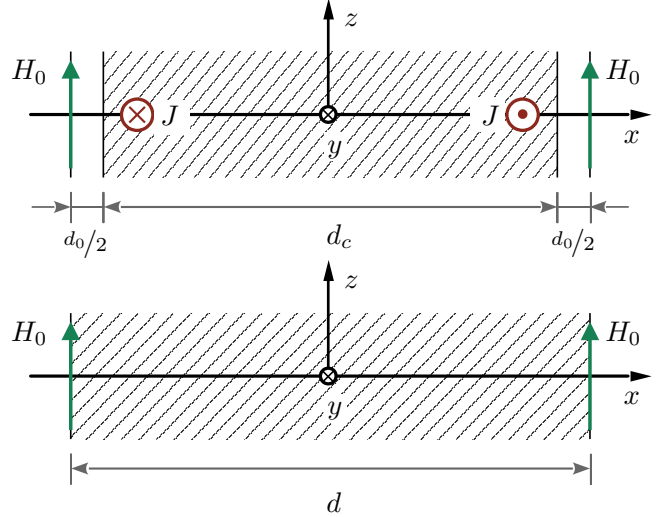


Fig. 1. The fully resolved cell problem (above) and the effective cell problem (below).

III. CELL PROBLEM AND EFFECTIVE MATERIAL

Consider a single steel sheet of thickness d_c with an insulation layer of thickness d_0 , aligned in parallel to the magnetic field. By assuming the sheet's length and width to be infinite, the edge effects vanish and the problem becomes one-dimensional. In this work the thickness of the sheet is assumed to be aligned with the x -axis. The magnetic field has a single component H_z , which induces laminar currents J_y , see Fig. 1. This so-called cell problem is excited by prescribing $H_z = H_0$ on the boundary for a given field strength H_0 .

The corresponding effective cell problem consists of a homogenized bulk material of thickness $d = d_c + d_0$, with its effective permeability μ_{eff} to be determined. Because the effective magnetic field $\mathbf{H}_{eff}$ is by necessity constant on the cell, currents cannot be directly represented and are included

by introducing an imaginary part to μ_{eff} .

The apparent power $S = P + jQ$ consists of the total losses P and the reactive power Q given by

$$P = \frac{1}{2} \int_{\Omega} \rho \mathbf{J} \cdot \mathbf{J}^* d\Omega, \quad Q = \frac{\omega}{2} \int_{\Omega} \mu \mathbf{H} \cdot \mathbf{H}^* d\Omega, \quad (3)$$

respectively. The effective permeability is chosen so that for the effective apparent power

$$S_{eff} = \frac{j\omega}{2} \int_{\Omega} \mu_{eff} \mathbf{H}_{eff} \cdot \mathbf{H}_{eff}^* d\Omega \quad (4)$$

there holds $S_{eff} = S$. For nonlinear materials, the effective permeability $\mu_{eff} = \mu_{eff}(H_0)$ is given by a complex nonlinear BH-curve. For more details see [2].

IV. THE EFFECTIVE FORMULATION

Let Ω_b be the bulk medium replacing the laminated core. For the effective material formulation, both a magnetic scalar potential Φ and a one-component current vector potential $\mathbf{T}$ in the direction perpendicular to the sheets are used. For a core aligned similar to the cell problem in Sec. III, the magnetic field is written as

$$\mathbf{H} = \mathbf{T} - \nabla \Phi + \mathbf{H}_{BS}, \quad \mathbf{T} = \begin{pmatrix} T_x \\ 0 \\ 0 \end{pmatrix}. \quad (5)$$

The idea is to use the effective material for the main magnetic field, and treat the stray fields, which cannot be described by the cell problem, separately.

The resulting weak formulation reads as: Find $T_x \in H^1(\Omega_b)$ and $\Phi \in H^1(\Omega)$ so that

$$\begin{aligned} & \int_{\Omega_0} j\omega \mu_0 \nabla \Phi \cdot \nabla q d\Omega \\ & + \int_{\Omega_b} \bar{\rho} \text{curl} \mathbf{T} \cdot \text{curl} \mathbf{v} + j\omega (\mathbf{T} - \nabla \Phi)^T \hat{\mu} (\mathbf{v} - \nabla q) d\Omega \\ & = \int_{\Omega_0} j\omega \mu_0 \mathbf{H}_{BS} \cdot \nabla q d\Omega - \int_{\Omega_b} j\omega \mathbf{H}_{BS}^T \hat{\mu} (\mathbf{v} - \nabla q) d\Omega \end{aligned} \quad (6)$$

for all $v_x \in H^1(\Omega_b)$ and $q \in H^1(\Omega)$. The anisotropic permeability tensor $\hat{\mu}$ and the averaged material parameters are given as

$$\hat{\mu} = \begin{pmatrix} \bar{\mu} & 0 & 0 \\ 0 & \mu_{eff} & 0 \\ 0 & 0 & \mu_{eff} \end{pmatrix}, \quad \bar{\mu} = \frac{\mu d_c + \mu_0 d_0}{d}, \quad \bar{\rho} = \frac{\rho d}{d_c}, \quad (7)$$

respectively. The apparent power of the effective solution can be evaluated by

$$S = \frac{1}{2} \int_{\Omega} j\omega \mathbf{H}^T \hat{\mu} \mathbf{H}^* + \bar{\rho} \text{curl} \mathbf{T} \cdot \text{curl} \mathbf{T}^* d\Omega. \quad (8)$$

V. NUMERICAL EXAMPLE

Consider a laminated open core consisting of 7 insulated steel sheets, surrounded in part by a cylindrical coil. The eddy current problem is solved using both a reference solution with $\mathbf{T}, \Phi - \Phi$ where the core is fully resolved in the mesh, and the effective material formulation where the core is treated as a homogenized bulk medium. Planes of symmetry are exploited.

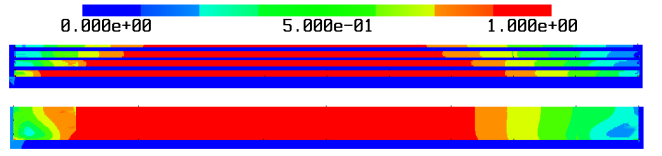


Fig. 2. Total magnetic flux $|\mathbf{B}|$ for the reference solution (above) and the effective solution (below, scaled by the fill factor for comparison) in T, rotated by 90° .

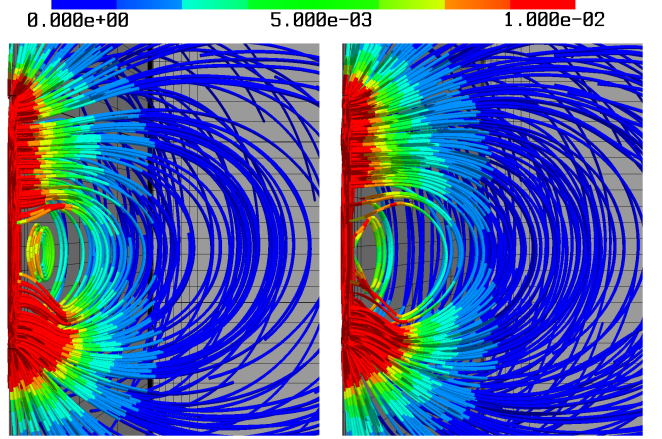


Fig. 3. Field lines of $\mathbf{B}$ for the reference solution (left) and the effective solution (right) in T.

The total magnetic flux $|\mathbf{B}|$ in a cross-section of the core for both solutions is shown in Fig. 2. It can be seen that the effective solution gives a good approximation of the reference field, homogenized over the core. Note that the effective solution is an average over both the sheets and the insulation layers, where the flux is negligible. For easier visual comparison, the effective solution has been rescaled by the inverse of the fill factor d_c/d .

The fields outside of the core are visualized in Fig. 3 via their field lines. It can be seen that the effective solution gives a good approximation of the magnetic flux everywhere.

The relative errors of the total losses and total reactive power obtained by the effective solution compared to the reference solution are 4.2% and 0.6%, respectively. For comparison: A conventional approach using only an effective material without accounting for stray fields shows errors of 18.7% and 1.7%, respectively.

VI. ACKNOWLEDGMENTS

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