

Homogenization of the Eddy Current Problem in Ferromagnetic Laminations Based on Numerical Simulations in Preprocessing

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Simulations of eddy currents in laminated ferromagnetic cores typically require a homogenization method. This paper introduces a numerical homogenization method based on computationally inexpensive simulations on well-constructed two-dimensional problems in the preprocessing phase. As a novelty, this method yields a complex-valued effective material that accounts for the nonlinearity of material properties and the edge effects of eddy currents, all without introducing additional degrees of freedom. The advantages of this approach are demonstrated through a comparison with an analytical homogenization procedure based on a one-dimensional approximation of eddy currents.

Index Terms—Eddy currents, homogenization, lamination, nonlinear magnetics, transformer cores, FEM

I. INTRODUCTION

FINITE element analysis of eddy currents in laminated ferromagnetic cores is computationally demanding. The thickness of a lamination sheet is significantly smaller than its length or width, resulting in ferromagnetic cores typically being composed of a large number of such sheets. Since air is permeable to magnetic flux, it is also necessary to include an air box in the model, meaning that the outer dimensions of the problem domain are much larger than the thickness of a single lamination sheet. Consequently, the number of finite elements required often exceeds the available computational resources. To enable the simulation of eddy currents in ferromagnetic lamination, an approximation of the eddy current problem must be introduced, significantly reducing the required finite element mesh density.

Using the well-known one-dimensional (1D) approximation of the eddy current problem, the eddy current density and magnetic flux density in a lamination sheet can be calculated from the average value of the magnetic flux density across the sheet's thickness [1]. Therefore, it is possible to homogenize the eddy current problem yielding an effective material. As a result, eddy currents can be accounted for during simulation by employing an element-wise averaged magnetic flux density on a coarse mesh [2], [3]. A multi-scale homogenization approach, based on the enrichment of problem variables with micro-shape functions, also accounts for the edge effects of eddy currents [4]. The calculation of effective material properties through numerical simulation of the 1D cell problem is shown in [5], [6]. Similarly, this paper presents a homogenization method for the eddy current problem based on fast two-dimensional (2D) simulations during preprocessing. The resulting effective material incorporates edge effects and nonlinearity.

II. HOMOGENIZATION PROCEDURE

The calculation of effective material properties is based on the 2D finite element method (FEM) within the laminated domain k_i , as shown in Fig. 1. Two types of elements k_i are distinguished: those with and without edge effects. Additionally, elements k_i vary in magnetic saturation, i.e.

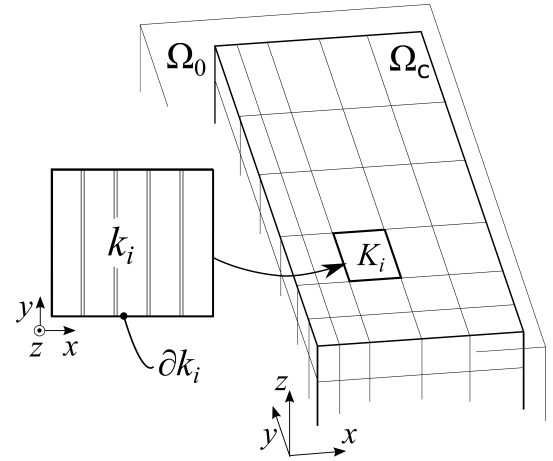


Fig. 1. The laminated 2D element k_i corresponds to the homogeneous 3D element K_i . The simulation in the preprocessing phase for the element k_i gives the effective material of the element K_i .

the average magnetic flux density value B_a , inherited from the corresponding three-dimensional (3D) element K_i during the simulation procedure. The 2D preprocessing simulations employ the nonlinear version of the $\mathbf{A}, T$ -formulation [7]:

$$\begin{aligned} \int_{k_i} \frac{j}{\omega\sigma} \nabla T \cdot \nabla T' dS - \int_{k_i} \nabla \times \mathbf{A} \cdot T' dS &= \int_{k_i} B_a \cdot T' dS \\ \int_{k_i} \nu_{fe}(|B|) \nabla \times \mathbf{A} \cdot \nabla \times \mathbf{A}' dS - \int_{k_i} T \cdot \nabla \times \mathbf{A}' dS &= 0, \end{aligned} \quad (1)$$

where $\mathbf{A}$ and T represent the magnetic vector potential and the one-component current potential, respectively. B_a is used as excitation, ω is the angular frequency, σ is the electrical conductivity and $\nu_{fe}(|B|)$ is the nonlinear magnetic reluctivity. From the solution of (1) then follow

$$B_{ez} = \nabla \times \mathbf{A} \quad (2)$$

$$\mathbf{J} = \nabla \times (T \mathbf{e}_z), \quad (3)$$

where B and $\mathbf{J}$ represent solutions for the magnetic flux density and the eddy currents density within 2D element k_i ,

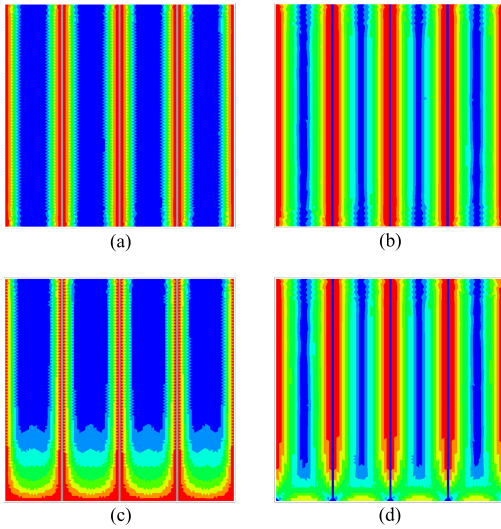


Fig. 2. Preprocessing simulation solutions for $B_a = 1.5$ T and $f = 50$ Hz. Subfigures (a) and (b) show absolute values of the magnetic flux density B and current density $\mathbf{J}$ in element k_1 , respectively. Subfigures (c) and (d) show absolute values of the magnetic flux density B and current density $\mathbf{J}$ in element k_2 , respectively.

respectively, and $\mathbf{e}_z$ represents the unit vector in z direction. It is evident that the vector fields $\mathbf{A}$ and $\mathbf{J}$ can have both x and y components. After B and $\mathbf{J}$ are obtained, the effective material, i.e. the homogenized magnetic reluctivity $\bar{\nu}_i$ is calculated from

$$\bar{\nu}_i(B_a) = \frac{1}{|k_i| B_a^2} \int_{k_i} (\nu_{fe}(|B|) B^2 - \frac{j}{\omega\sigma} \mathbf{J} \cdot \mathbf{J}) dS \quad (4)$$

In Fig. 2, the simulation results for B and $\mathbf{J}$ are presented for two different elements, k_1 and k_2 . Element k_2 includes edge effects, resulting in different effective material properties compared to element k_1 . Since ν_{k_i} is nonlinearly dependent on $|B|$, as shown in Fig. 3, the nonlinear 2D simulation is carried out for various values of B_a .

III. COMPARISON OF HOMOGENIZATION METHODS

To illustrate benefits of the presented numerical homogenization method, a comparison is made to the analytical homogenization procedure based on the 1D approximation of the eddy current problem [2]. For a lamination sheet of thickness d analytical homogenization yields an effective material, i.e. homogenized magnetic reluctivity

$$\bar{\nu}_i(B_a) = \frac{\omega\sigma d(1+j)\delta(B_a)}{8 \left(\sinh \left(\frac{(1+j)d}{2\delta(B_a)} \right) \right)^2} \cdot \sinh \left(\frac{(1+j)d}{\delta(B_a)} \right) \quad (5)$$

where $\delta(B_a)$ is given by

$$\delta(B_a) = \sqrt{\frac{2\nu_{fe}(B_a)}{\omega\sigma}} \quad (6)$$

In (6), ν_{fe} depends nonlinearly on B_a , as shown in Fig. 3. Therefore, $\bar{\nu}_i$ must be calculated for different values of B_a . Thus, (5) is a linear model embedded within a nonlinear procedure.

Table I shows a comparison of homogenized reluctivities obtained through numerical and analytical homogenization

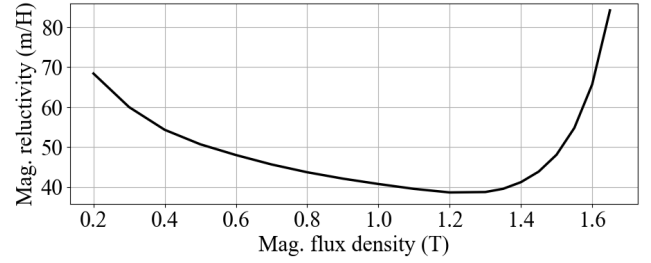


Fig. 3. Magnetic reluctivity ν_{fe} as a function of the magnetic flux density

TABLE I
HOMOGENIZED RELUCTIVITY

Finite element	Mag. flux density B_a (T)	Frequency f (Hz)	Analytically homogenized reluctivity $\bar{\nu}_i$ (m/H)	Numerically homogenized reluctivity $\bar{\nu}_i$ (m/H)
k_1	1	50	$43.8 + j13.4$	$43.6 + j13.3$
	1	150	$50.0 + j38.5$	$49.8 + j38.3$
	1.5	50	$62.6 + j13.4$	$64.1 + j13.8$
	1.5	150	$67.1 + j39.5$	$78.1 + j46.1$
k_2	1	50	$43.8 + j13.4$	$43.7 + j12.2$
	1	150	$50.0 + j38.5$	$50.6 + j36.2$
	1.5	50	$62.6 + j13.4$	$63.5 + j12.5$
	1.5	150	$67.1 + j39.5$	$73.6 + j39.7$

Finite element k_1 is free of edge effects, whereas k_2 has edge effects.

methods for $d = 0.5$ mm, $\sigma = 2 \cdot 10^6$ S/m and nonlinear ν_{fe} shown in Fig. 3. For element k_1 , numerical and analytical $\bar{\nu}_i$ agree well if the magnetic saturation is not pronounced. Otherwise, the numerical approach is needed, especially for higher frequencies, where δ approaches $d/2$. As can be seen in Fig. 3, magnetic saturation effects occur for $B_a = 1.5$ T. If edge effects are present, as is the case for element k_2 , a noticeable difference between numerical and analytical $\bar{\nu}_i$ is present regardless of magnetic saturation and frequency. In the full paper, this effective material will be applied in the three-dimensional simulation of the eddy current problem, enabling the use of a much coarser mesh compared to the reference model.

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