

Efficient finite element models for Milliken conductors using homogenization techniques based on helicoidal symmetry

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Milliken conductors are a widely used design for the inner conductors of AC high-voltage power cables. Determining their AC resistance numerically is a challenge due to the inherent multiscale nature of the problem, involving the strand diameter, the pitch length of strand layers, and the pitch length of the segments. To address this, we propose a finite element model that takes advantage of homogenization techniques based on helicoidally symmetric boundary value problems.

Index Terms—coordinate transformations, dimension reduction, eddy currents, finite elements, homogenization, Milliken conductors, proximity effect, segmented conductors, skin effect

I. INTRODUCTION

HIGH-VOLTAGE power cables with large cross-sections (e.g., 3000 mm² copper cross-section) are commonly used for transmitting substantial amounts of energy. In alternating current (AC) operation, the skin effect significantly limits the efficient use of a solid inner conductor, even at relatively low frequencies like 50 Hz. Consequently, these inner conductors are often designed as segmented or Milliken conductors (see Fig. 1). The total cross-section is typically divided into five to six electrically insulated segments, which are twisted around the conductor's core. Each segment is further composed of insulated strands arranged in layers, which are also twisted (around the segment core though) with variations in twist pitch and possibly direction.

Accurately determining the AC resistance of such conductors is challenging, both numerically – due to the multiscale nature of the problem – and experimentally, as no standardized measurement method currently exists. This lack of standardization results in significant discrepancies in measurement results as recently shown in a round robin test [1]. While approximate formulas have been developed in recent years to estimate the AC resistance, these often fail to account for critical design parameters, such as the number of segments or variations in strand layer pitch lengths [2], [3]. Moreover, some of these formulas are empirically adjusted to match measured resistance values, which is problematic given the absence of standardized measurement procedures. To address this gap, our research focuses on developing computationally efficient finite element models for Milliken conductors. By leveraging homogenization techniques based on helicoidally symmetric boundary value problems (BVPs), we aim to substantially reduce computational costs compared to full-scale 3D models.

II. HOMOGENIZATION STRATEGY

As previously mentioned, the geometry of a Milliken conductor results in a 3-D multiscale problem when modeling its electromagnetic behavior via eddy current BVPs (ECBVPs). Neglecting the twists to simplify the problem into a 2-D one not only leads to inaccuracies in calculating ohmic power



Fig. 1. Milliken conductor of an XLPE insulated high-voltage power cable.

losses [4], but also eliminates the ability to analyze crucial design parameters in detail.

When applicable, an elegant approach to reduce computational costs is to take advantage of the symmetries inherent in a problem. In our previous work [6], we demonstrated how helicoidally symmetric ECBVPs can be solved equivalently in 2-D. Here, the core idea is to introduce a helicoidal coordinate system (u, v, w) in which a uniformly twisted structure appears straight (see Fig. 2).

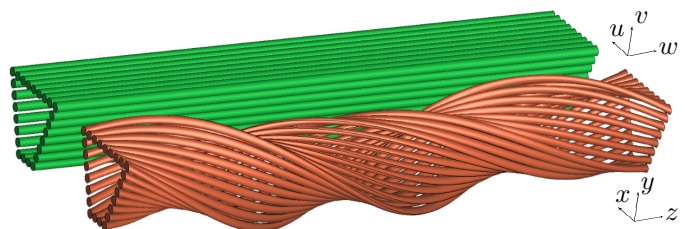


Fig. 2. Geometric representation of twisted strands. From back to front: Representation in helicoidal coords (u, v, w) colored in green (w -invariant) and the same object expressed in Cartesian coords (x, y, z) colored in red. Modeled using CST Studio Suite [5].

Unfortunately, this transformation method cannot be directly applied to Milliken conductors due to the presence of two distinct twist levels: (a) the twist of the segments around the conductor core, characterized by the pitch length p_s , and (b) the

twist of the i -th strand layer around its respective segment core, characterized by the pitch length $p_{l,i \in N_l}$, where N_l denotes the number of strand layers within a segment (typically 5 or 6).

To address this, we treat the two twist levels sequentially: The key idea is to encapsulate the effects of the twist at the strand layer level into homogenized material properties (layer per layer). These homogenized properties are then used in a subsequent problem to model the strand layers as if they were solid materials at the segment twist level, i.e., geometrical details are not considered anymore, s.t., only the twist of the segments needs to be taken into account.

For homogenizing the strand layers we adopt the approach outlined in [7]. In summary, this method ensures that the averaged apparent power remains invariant between the homogenized model and the original model. This is achieved by introducing a modified, complex-valued reluctivity tensor ν_{prox} , which accounts for losses caused by the proximity effect. Additionally, an external impedance Z_{skin} is introduced and coupled to the field problem to represent losses arising from the skin effect. To separately evaluate the contributions of the skin and proximity effects, the cell problem (minimal original problem) consisting of a single layer of twisted strands (see Fig. 2) is excited as follows:

- Skin effect excitation (SEE): A fixed total current is applied into each twisted strand at a specified frequency.
- Proximity effect excitation (PEE): The twisted strand layer is exposed to uniform (and time-harmonic) external magnetic fields, while maintaining zero total current in each strand. Here we distinguish between the two cases in which the external field is directed transversely (PETE) and longitudinally (PELE).

The excitation modes SEE & PELE can be solved by the $\mathbf{H} - \phi$ finite element formulation already presented in [6]. Note that for the PEE case, no cohomology basis functions are required, as the circulation of the magnetic field is constrained to be zero. This means that the magnetic field in the insulating domain can be exactly expressed as the gradient of a scalar potential. However, the formulation needs to be extended for the excitation mode PETE. Here, the magnetic field $\mathbf{H}_{uvw}$ fully depends on three variables u, v, w in the space Ω_{uvw} , i.e., the computation cannot be directly reduced equivalently to a 2-D problem. Intuitively speaking, this is because the transverse external field lacks helicoidal symmetry (see Fig. 3).¹ Since the geometry is periodically w.r.t. the pitch length $p_{l,i \in N_l}$ though, the following separation of variables and spatial Fourier series is possible [8]:

$$\mathbf{H}_{uvw}(u, v, w) = \sum_{k=-\infty}^{\infty} \mathbf{H}_{uvw,k}(u, v) f_k(w), \quad (1)$$

with modes $f_{k<0} = \sqrt{2} \cos(kw/\beta)$, $f_{k>0} = \sqrt{2} \sin(kw/\beta)$ and $f_{k=0} = 1$, where $\beta = p_{l,i \in N_l}/2\pi$. It can be shown, that for the case of a uniform transversally directed external magnetic

¹The external magnetic field in terms of helicoidal coords is given by: $\mathbf{H}_{uvw,\text{ext}}(u, v, w) = J_{\phi-1}^T(u, v, w) \mathbf{H}_{xyz,\text{ext}}(x, y, z)$, resp. $J_{\phi-1}^T(u, v, w) [H_0, 0, 0]^T$, in case of a x -directed external field. $J_{\phi-1}$ denotes the Jacobian of the coordinate transformation $\phi^{-1} : \Omega_{uvw} \rightarrow \Omega_{xyz}$ and is dependent on u, v, w [8].

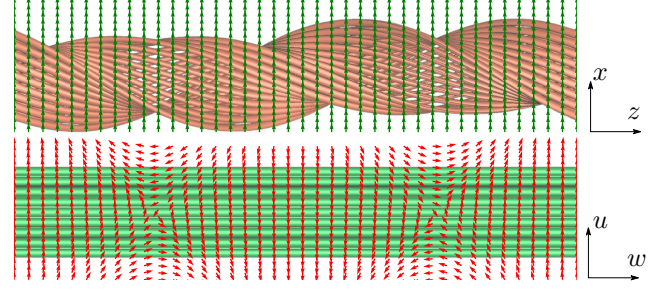


Fig. 3. Bird's eye view of the arrangement in Fig. 2 for $y = \text{const.}$: x -directed external magnetic field (top) transforms into a non-uniform vector field in helicoidal coords (normalized, bottom plot). Note its w -periodicity with periodic length $p_{l,i \in N_l}$ (pitch length of the specific layer) though.

field (in Cartesian coords) only the modes $k = \pm 1$ are non-zero [8]. Embedding this idea into FE formulation [6] enables to treat the w -dependency analytically and hence to solve the PETE mode in a quasi 3-D manner. A preliminary result of our GetDP [9] implementation is shown in Fig. 4. The full paper will present a detailed analysis of our proposed method including comparative 3-D calculations.

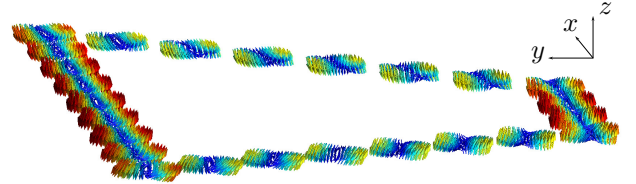


Fig. 4. Real part of the current density at plane $z = 0$ caused by a time-harmonic (50 Hz), x -directed uniform external magnetic field penetrating the strands. Colorcoding w.r.t. $|\text{Re } \mathbf{J}_{xyz}|$ from low (blue) to high values (red).

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